Chapter 2. Mathematical Models of System

- Differential Equations of Physical System
- Linear Approximations of Physical System
- The Laplace Transform
- The Transfer Function Of Linear System
- Block Diagram Models
- Signal-Flow Graph Model
- Computer Analysis of control system
- Design Examples
- The Simulation of System Using Matlab
Differential Equations of Physical System

The approach to dynamic system problem can be listed as follow:

1. Define the system and its components
2. Formulate the mathematical model and list the necessary assumptions.
3. Write the differential equations describing the model.
4. Solve the equations for the desired output variables.
5. Examine the solutions and the assumptions.
6. If necessary, reanalyze or redesign the system
FIGURE 2.1 (a) Torsional spring-mass system. (b) Spring element.
\[ F_M(t) = Ma(t) \]
\[ F_b(t) = Bv(t) \]
\[ F_k(t) = ky(t) \]

\[ v(t) = \frac{dy(t)}{dt} \quad a(t) = \frac{dv(t)}{dt} \]
\[ y(t) = \int v(t)\,dt \quad , \quad v(t) = \int a(t)\,dt \]

\[ F_M(t) + F_b(t) + F_k(t) = r(t) \]
\[ M \frac{d^2y(t)}{dt^2} + b \frac{dy(t)}{dt} + ky(t) = r(t) \]
The Transfer Function (T.F.) of Linear Systems

Definition:

The ratio of the Laplace transform of the output variable to the Laplace transform of the input variable, with all initial conditions assumed to be zero.

The Laplace transform of the impulse response, with all the initial conditions set to zero.

\[ \frac{Y(s)}{R(s)} = G(s) \]

or

\[ G(s) = L[y(t)] \]
\[ f_M(t) = Ma(t) \quad f_B(t) = Bv(t) \quad f_k(t) = Kx(t) \]

\[ f(t) = f_M(t) + f_B(t) + f_K(t) \]

\[ f(t) = Ma(t) + Bv(t) + Kx(t) \]

\[ x(t) = \int v(t) dt \quad a(t) = \frac{d}{dt} v(t) \]

\[ f(t) = M \frac{d}{dt} v(t) + Bv(t) + K \int v(t) dt \]

Take Laplace and set I.C. = 0

\[ F(s) = MsV(s) + BV(s) + K \frac{1}{s} V(s) = \left[ Ms + B + \frac{K}{s} \right] V(s) \]

\[ \frac{V(s)}{F(s)} = \frac{1}{Ms + B + \frac{K}{s}} \]
\[ r(t) = i_R(t) + i_L(t) + i_C(t) \]

\[ r(t) = \frac{v_o(t)}{R} + \frac{1}{L} \int v_o(t) \, dt + C \frac{d}{dt} v_o(t) \]

Take Laplace and set I.C. = 0

\[ R(s) = \frac{V_o(s)}{R} + \frac{1}{L} \frac{1}{s} V_o(s) + CsV_o(s) = \left[ \frac{1}{R} + \frac{1}{Ls} + Cs \right] V_o(s) \]

\[ \frac{V_o(s)}{R(s)} = \frac{1}{Cs + \frac{1}{R} + \frac{1}{Ls}} \quad \quad \frac{V(s)}{F(s)} = \frac{1}{Ms + B + \frac{K}{s}} \]

Analogous variables

if \( M = C, \quad B = \frac{1}{R}, \quad K = \frac{1}{L} \)
<table>
<thead>
<tr>
<th>Type of Element</th>
<th>Physical Element</th>
<th>Describing Equation</th>
<th>Energy $E$ or Power $\mathcal{P}$</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inductive storage</td>
<td>Electrical inductance</td>
<td>$v_{21} = L \frac{di}{dt}$</td>
<td>$E = \frac{1}{2} Li^2$</td>
<td><img src="image1" alt="Electrical Inductance" /></td>
</tr>
<tr>
<td></td>
<td>Translational spring</td>
<td>$v_{21} = \frac{1}{k} \frac{dF}{dt}$</td>
<td>$E = \frac{1}{2} \frac{F^2}{k}$</td>
<td><img src="image2" alt="Translational Spring" /></td>
</tr>
<tr>
<td></td>
<td>Rotational spring</td>
<td>$\omega_{21} = \frac{1}{k} \frac{dT}{dt}$</td>
<td>$E = \frac{1}{2} \frac{T^2}{k}$</td>
<td><img src="image3" alt="Rotational Spring" /></td>
</tr>
<tr>
<td></td>
<td>Fluid inertia</td>
<td>$p_{21} = \frac{1}{k} \frac{dQ}{dt}$</td>
<td>$E = \frac{1}{2} IQ^2$</td>
<td><img src="image4" alt="Fluid Inertia" /></td>
</tr>
<tr>
<td>Capacitive storage</td>
<td>Electrical capacitance</td>
<td>$i = C \frac{dv_{21}}{dt}$</td>
<td>$E = \frac{1}{2} Cv_{21}^2$</td>
<td><img src="image5" alt="Electrical Capacitance" /></td>
</tr>
<tr>
<td></td>
<td>Translational mass</td>
<td>$F = M \frac{dv_2}{dt}$</td>
<td>$E = \frac{1}{2} Mv_2^2$</td>
<td><img src="image6" alt="Translational Mass" /></td>
</tr>
<tr>
<td></td>
<td>Rotational mass</td>
<td>$T = J \frac{d\omega_2}{dt}$</td>
<td>$E = \frac{1}{2} J\omega_2^2$</td>
<td><img src="image7" alt="Rotational Mass" /></td>
</tr>
<tr>
<td></td>
<td>Fluid capacitance</td>
<td>$Q = C_f \frac{dP_{21}}{dt}$</td>
<td>$E = \frac{1}{2} C_f P_{21}^2$</td>
<td><img src="image8" alt="Fluid Capacitance" /></td>
</tr>
<tr>
<td></td>
<td>Thermal capacitance</td>
<td>$q = C_t \frac{dT_2}{dt}$</td>
<td>$E = C_t T_2$</td>
<td><img src="image9" alt="Thermal Capacitance" /></td>
</tr>
<tr>
<td></td>
<td>Electrical resistance</td>
<td>$i = \frac{1}{R} v_{21}$</td>
<td>$\mathcal{P} = \frac{1}{R} v_{21}^2$</td>
<td><img src="image10" alt="Electrical Resistance" /></td>
</tr>
<tr>
<td>Energy dissipators</td>
<td>Translational damper</td>
<td>$F = bv_{21}$</td>
<td>$\mathcal{P} = bv_{21}^2$</td>
<td><img src="image11" alt="Translational Damper" /></td>
</tr>
<tr>
<td></td>
<td>Rotational damper</td>
<td>$T = b\omega_{21}$</td>
<td>$\mathcal{P} = b\omega_{21}^2$</td>
<td><img src="image12" alt="Rotational Damper" /></td>
</tr>
<tr>
<td></td>
<td>Fluid resistance</td>
<td>$Q = \frac{1}{R_f} P_{21}$</td>
<td>$\mathcal{P} = \frac{1}{R_f} P_{21}^2$</td>
<td><img src="image13" alt="Fluid Resistance" /></td>
</tr>
<tr>
<td></td>
<td>Thermal resistance</td>
<td>$q = \frac{1}{R_t} T_{21}$</td>
<td>$\mathcal{P} = \frac{1}{R_t} T_{21}$</td>
<td><img src="image14" alt="Thermal Resistance" /></td>
</tr>
</tbody>
</table>
© Rotational Motion

1. Inertia

Newton’s Law for rotation motion:

\[ \sum T(t) = J \alpha(t) = J \frac{d}{dt} \omega(t) = J \frac{d^2}{dt^2} \theta(t) \]

- \( T \): Torque
- \( J \): Inertia
- \( \alpha \): Angular acceleration
- \( \omega \): Angular velocity
- \( \theta \): Angular displacement

2. Torsional Spring

\[ T(t) = k \theta(t) \]

- \( k \): Torsional spring constant

3. Friction for rotational motion: viscous, static, coulomb friction

\[ T(t) = B \frac{d}{dt} \theta(t) \]

- \( B \): Viscous friction coefficient
• **DC motor**
• **Field-current controlled DC motor**
• **Armature-controlled DC motor**

Mathematical modeling of PM DC motor:

The air-gap flux of the dc motor is

$$\phi(t) = k_f i_f(t)$$

and the torque developed by the dc motor is

$$T_m(t) = k_1 \phi(t) i_a(t) = k_1 k_f i_f(t) i_a(t)$$

$$= k_1 k_f I_f i_a(t), \quad i_f(t) = \text{constant}$$

$$= k_m i_a(t)$$
\[ v_a(t) = R_a i_a(t) + L_a \frac{d}{dt} i_a(t) + e_b(t) \quad e_b(t) = k_b \omega(t) \quad T_m(t) = k_m i_a(t) \]

\[ T_L = J \frac{d}{dt} \omega(t) + B \omega(t) \quad T_m(t) = T_d(t) + T_L(t) \]

T.L & I.C.=0

\[
\frac{V_a(s) - E_b(s)}{R_a + L_a s} = I_a(s) \quad (1) \quad T_m(s) = k_m I_a(s) \quad (2)
\]

\[
\frac{T_L(s)}{J s + B} = \omega(s) \quad (3) \quad E_b(s) = k_b \omega(s) \quad (4)
\]

\[ V_a(s) \quad + \quad \frac{1}{R_a + L_a s} \quad I_a(s) \quad + \quad \frac{1}{J s + B} \quad \omega(s) \]

\[ e_b(s) \quad - \quad T_m(s) \quad k_m \quad T_d(s) \quad \]

\[ k_b \quad \]
The cause and effect equation for the motor circuit are

\[ v_a(t) = R_a i_a(t) + L_a \frac{d}{dt} i_a(t) + e_b(t) \]

\[ e_b(t) = k_b \omega(t) = k_b \frac{d}{dt} \theta(t) \]

\[ T(t) = J \frac{d}{dt} \omega(t) + B \omega(t) + T_d \]

\[ = J \frac{d^2}{dt^2} \theta(t) + B \frac{d}{dt} \theta(t) + T_d \]

The motor torque is equal to the torque delivered to the load, that is

\[ T_m(t) = T(t) \]

Hence the transfer function of the dc motor, with \( T_d = 0 \), is

\[ \frac{\omega(s)}{V_a(s)} = \frac{k_m}{(R_a + sL_a)(Js + B) + k_b k_m} \]
If \( L_a/R_a \approx 0 \), the approximate model of the dc motor is obtained as

\[
\frac{\omega(s)}{V_a(s)} = \frac{k_m}{JRa_s + (BR_a + k_b k_m)}
\]

\[
\Delta = \frac{k}{s\tau + 1}
\]

where

\[
\tau = \frac{JR_a}{BR_a + k_b k_m}, \quad k = \frac{k_m}{BR_a + k_b k_m}
\]

**Relation between** \( k_m \) **and** \( k_b \)**

The power input to the rotor = The power delivered to the shaft

\[
e_b(t)i_a(t) = T(t)\omega(t)
\]

\( \Leftrightarrow \)

\[
k_b\omega(t)i_a(t) = k_m i_a(t)\omega(t)
\]

\( \Leftrightarrow k_m = k_b \)
Homework 1:

\[ \frac{X_1(s)}{F(s)} = ? \]
DC motor Field-controlled

Home work 2:

\[ \frac{\omega(s)}{V_f(s)} = ? \]

Table 2.5

Transfer Functions of Dynamic Elements and Networks
5. dc motor, field-controlled, rotational actuator
\[ \frac{V_2(s)}{V_1(s)} = ? \]

Homework 3

Table 2.5

Transfer Functions of Dynamic Elements and Networks
4. Lead-lag filter circuit
Homework 4:

\[ \theta_L = n \theta_m \]

\[ T_m = J \frac{d}{dt} \omega_m + B \omega_m, J = ?, B = ? \]

\[ T_L = J_L \frac{d}{dt} \omega_L + B_L \omega_L \]

Table 2.5

Transfer Functions of Dynamic Elements and Networks
10. Gear train, rotational transformer
Armature Control

Constant Torque

Field Control

Constant Power

\[ P = \omega \tau \]
• Tachometer

\[ e_t(t) = k_t \omega(t) = k_t \frac{d}{dt} \theta(t) \]

• Potentiometer

\[ e_p(t) = k_p \theta(t), \quad k_p = \frac{V_{app}}{2\pi N} \]
Linear Approximations of Physical Systems

A linear system satisfied the properties of superposition and homogeneity.

\[ r(t) \rightarrow y(t) \iff y(t) = g[r(t)] \]

\[
\begin{align*}
  r_1(t) &\rightarrow y_1(t) \\
  r_2(t) &\rightarrow y_2(t)
\end{align*}
\]

\[
\Rightarrow r(t) = \alpha r_1(t) + \beta r_2(t) \rightarrow y(t) = \alpha y_1(t) + \beta y_2(t)
\]

\[ g[\alpha r_1(t) + \beta r_2(t)] = \alpha g[r_1(t)] + \beta g[r_2(t)] \]
FIGURE 2.5

(a) A mass sitting on a nonlinear spring. 
(b) The spring force versus $y$. 
Linearization of a nonlinear systems:

\[ y(t) = g[x(t)] \]

Expanding the nonlinear equation into a Taylor series about the operation point, then we have

\[
y = g(x) = g(x_o) + \frac{d}{dx} g(x) \bigg|_{x=x_o} \frac{(x-x_o)}{1!} \frac{d^2}{dx^2} g(x) \bigg|_{x=x_o} \frac{(x-x_o)^2}{2!} + \ldots
\]

Neglecting all the high order terms, to yield

\[
y = g(x_o) + \frac{d}{dx} g(x) \bigg|_{x=x_o} \frac{(x-x_o)}{1!} = y_0 + m \cdot (x - x_o)
\]

\[ \Rightarrow y - y_0 = m \cdot (x - x_o) \]

or \[ \Delta y = m \cdot \Delta x \]

\[
y = g(x_1, \cdots, x_n)
\]

\[
= g(x_{1o}, \cdots, x_{no}) + \frac{\partial g}{\partial x_1} \bigg|_{x=x_o} (x_1 - x_{1o}) + \frac{\partial g}{\partial x_2} \bigg|_{x=x_o} (x_2 - x_{2o}) + \cdots + \frac{\partial g}{\partial x_n} \bigg|_{x=x_o} (x_n - x_{no})
\]
Example 2.1 Pendulum oscillator model

\[ T = Mgl \sin \theta \]

\[ T - T_0 \approx Mgl \frac{\partial \sin \theta}{\partial \theta} \bigg|_{\theta = \theta_0} (\theta - \theta_0) \]

where \( T_0 = 0 \).

\[ T = Mgl(\cos 0^\circ)(\theta - 0) \]

\[ = Mgl\theta \]
© First Order Systems

\[ v_i(t) = v_r(t) + v_c(t) = v_r(t) + v_o(t) \]

\[ v_r(t) = Ri(t), v_o(t) = \frac{1}{C} \int_0^t i(t) dt \Rightarrow i(t) = C \frac{d v_o(t)}{d t} \]

\[ v_i(t) = RC \frac{d v_o(t)}{d t} + v_o(t) \Rightarrow V_i(s) = (RCs + 1)V_o(s) \]

\[ \frac{V_o(s)}{V_i(s)} = \frac{1}{RCs + 1} = \frac{a}{s + a}, \quad a = \frac{1}{RC} \]

\[ v_i(t) = \begin{cases} 0, & t < 0 \\ A, & t > 0 \end{cases} \]

\[ V_o(s) = \frac{A}{s} \cdot \frac{a}{s + a} = A \frac{1}{s} - \frac{1}{s + a} \]

\[ L^{-1}\{V_o(s)\} = V_o(t) = A(1 - e^{-at}) = A(1 - e^{-\frac{t}{\tau}}) \]

**Definition:** The time constant \( \tau \) for the first-order system

\[ \text{Time Constant} = \tau = \frac{1}{a} \]
First-order step response

\[ t = \tau, \quad v_o(t) = A(1 - e^{-1}) = 0.632A \]

\[ t = 2\tau \quad v_o(t) = A(1 - e^{-2}) = 0.864A \]

\[ t = 3\tau \quad v_o(t) = A(1 - e^{-3}) = 0.95A \]

\[ t = 4\tau \quad v_o(t) = A(1 - e^{-4}) = 0.981A \]

\[ t = 5\tau \quad v_o(t) = A(1 - e^{-5}) = 0.993A \]

\[ \left. \frac{dv_o(t)}{dt} \right|_{t=0} = ? \]

**Definition:** The settling time \( (t_s) \) for the first-order system

\[ t_s = 5\tau \]

---

**gain-time constant form**

\[ \frac{10}{6s+1}, \, \text{gain} = k = 10, \, \tau = 6 \]

**pole-zero form**

\[ \frac{10}{6s + \frac{1}{6}}, \, \text{pole} = -\frac{1}{6} \]
**System Parameters Identification**

\[ r(t) = \begin{cases} 0, & t < 0 \\ A, & t > 0 \end{cases} \Rightarrow L\{r(t)\} = R(s) = \frac{A}{s} \]

\[ G(s) = \frac{k}{s + a} \]

\[ Y(s) = R(s) \cdot G(s) = \frac{A}{s} \cdot \frac{k}{s + a} = \frac{Ak}{a} \left( \frac{1}{s} - \frac{1}{s + a} \right) \]

\[ L^{-1}\{Y(s)\} = y(t) = \frac{Ak}{a} \left(1 - e^{-\alpha t}\right), \tau = \frac{1}{a} \]

\[ y(\tau) = 0.636 y(\infty) = y_{ss}, y_{ss} = \frac{Ak}{a} \]

\[ k = \frac{y_{ss} \cdot a}{A} = 1.59 \]

\[ A = 3, y_{ss} = 2.5, \tau = \frac{1}{a} = 0.2 \Rightarrow a = 5, k = \frac{y_{ss} \cdot a}{A} = \frac{2.5 \cdot 5}{3} = 4.1667 \]
Start simulink tools box

Matlab Command Windows

To get started, select "MATLAB Help" from the Help menu.
**Transfer Func.** Matrix expression for numerator, vector expression for denominator, equals the number of rows in the numerator. Coefficients are for descending powers of s.

- Simulink Library Browser
  - Continuous
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  - Model Verification
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  - Ports & Subsystems
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  - User-Defined Functions
  - Aerospace Blockset
  - CDMA Reference Blockset
  - Communications Blockset
  - Control System Toolbox
  - DSP Blockset
  - Dials & Gauges Blockset
  - Embedded Target for Motorola MPC5
  - Embedded Target for TI C6000 DSP
  - Fixed-Point Blockset
  - Fuzzy Logic Toolbox

Diagram:
- Step
- Transfer Func
- Scope

Math expressions:
- \( \frac{du}{dt} \) Derivative
- \( \frac{1}{s} \) Integrator
- \( x = Ax + Bu \)
- \( y = Cx + Du \) State-Space
- \( \frac{1}{s+1} \) Transfer Func
- Transport Delay
- Variable Transport Delay
- Zero-Pole

Graph:
- Time axis from 0 to 2
- Y-axis from 0 to 3
plot(y(:,1),y(:,2),y(:,1),y(:,3))

First-order step response

a = 5
k = 4.167
**S-plane and poles of control system**

\[ F(s) = s + a = 0, \quad \text{Characteristic equation} \]

\[ s = -a \quad \text{Pole of the first-order system} \]

Response with initial conditions (free response)

\[ y_0(t) = y(0)e^{-at} = y(0)e^{-\frac{t}{\tau}} \]

The model parameter \( k \) has an effect on the steady state of the output, but no effect on the shape of the response. A large negative pole yields a small time constant that the transition response will decay quickly.
<table>
<thead>
<tr>
<th>Transformation</th>
<th>Original Diagram</th>
<th>Equivalent Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Combining blocks in cascade</td>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td>2. Moving a summing point behind a block</td>
<td><img src="image3" alt="Diagram" /></td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
<tr>
<td>3. Moving a pickoff point ahead of a block</td>
<td><img src="image5" alt="Diagram" /></td>
<td><img src="image6" alt="Diagram" /></td>
</tr>
<tr>
<td>4. Moving a pickoff point behind a block</td>
<td><img src="image7" alt="Diagram" /></td>
<td><img src="image8" alt="Diagram" /></td>
</tr>
<tr>
<td>5. Moving a summing point ahead of a block</td>
<td><img src="image9" alt="Diagram" /></td>
<td><img src="image10" alt="Diagram" /></td>
</tr>
<tr>
<td>6. Eliminating a feedback loop</td>
<td><img src="image11" alt="Diagram" /></td>
<td><img src="image12" alt="Diagram" /></td>
</tr>
</tbody>
</table>
\[ C(s) = X(s)G(s) \]

\[ X(s) = R(s) - B(s) \]

\[ B(s) = C(s)H(s) \]

\[ C(s) = (R(s) - B(s))G(s) \]

\[ = (R(s) - C(s)H(s))G(s) \]

\[ C(s)[1 + H(s)G(s)] = R(s)G(s) \]

\[ \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \]
The block diagram

FIGURE 2.26 Multiple-loop feedback control system.
Transfer function:

\[ T(s) = \frac{Y(s)}{R(s)} = \frac{b}{s + (a + b)} \]

Output:

\[ Y(s) = T(s) \cdot R(s) = \frac{b}{s + (a + b)} \cdot \frac{A}{s} \]

\[ y(t) = L^{-1}\{Y(s)\} = \frac{Ab}{a + b} (1 - e^{-(a+b)t}), \quad \forall t \geq 0 \]

Time constant:

\[ \tau = \frac{1}{a + b} \]

Error signal:

\[ E(s) = R(s) - Y(s) = \frac{As + Aa}{s(s + a + b)} \]

\[ e(t) = L^{-1}\{E(s)\} = \frac{Aa}{a + b} + \frac{Ab}{a + b} e^{-(a+b)t} \]
HOME WORK 5

1. \( R(s) = \frac{2}{s}, a=5, b=4 \)
2. \( R(s) = \frac{1}{s}, a=2, b=-4 \)
3. \( R(s) = \frac{3}{s}, a=-2, b=4 \)

\( C(t) =? \)

Plot the waveform of \( C(t) \), marked the final value and time constant

HOME WORK 6

1. \( R(s) = \frac{2}{s}, a=5, b=4 \)
2. \( R(s) = \frac{1}{s}, a=2, b=-4 \)
3. \( R(s) = \frac{3}{s}, a=-2, b=4 \)

\( C(t) =? \)

Plot the waveform of \( C(t) \), marked the final value and time constant
HOME WORK 7

\[ R(s) + \frac{b}{s + a} \rightarrow C(s) \]

Graphs:

1. \( r(t) = 1 \) and \( c(t) = 1.5 \)
   - Time: 0.25 to 1 s
   - Output: 0.95 to 1.5

2. \( r(t) = 1 \) and \( c(t) = 1.6 \)
   - Time: 30 ms
   - Output: 1.01 to 1.6

3. \( r(t) = 2 \) and \( c(t) = 2 \)
   - Time: 0.5 to 2 s
   - Output: -2 to 2

Questions:

(1) \( a = ? \) \( b = ? \)
(2) \( a = ? \) \( b = ? \)
(3) \( a = ? \) \( b = ? \)
FIGURE 2.28  Signal-flow graph of the dc motor.
Signal-flow Graph Modes

- **Branch and Branch Gain:**

- **Nodes:**

- **Forward-path and Forward-path gain:**

- **Loop and Loop gain:**

- **Non-touching:**

Mason’s signal-flow gain formula:

\[
T_{ij} = \frac{\sum_{k=1}^{N} P_{ijk} \Delta_{ijk}}{\Delta} = \frac{\text{Output node}}{\text{input node}}
\]

- \( P_{ijk} \):
- \( \Delta \):
- \( \Delta_{ijk} \):
\[ L_1^{1,2} = G_2 H_2 \quad L_1^{1,2} L_3^{4,5} = G_2 H_2 G_6 H_6 \]
\[ L_2^{2,3} = G_3 H_3 \quad L_1^{1,2} L_4^{5,6} = G_2 H_2 G_7 H_7 \]
\[ L_3^{4,5} = G_6 H_6 \quad L_2^{2,3} L_3^{4,5} = G_3 H_3 G_6 H_6 \]
\[ L_4^{5,6} = G_7 H_7 \quad L_2^{2,3} L_4^{5,6} = G_3 H_3 G_7 H_7 \]
\[ \Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_3 + L_1 L_4 + L_2 L_3 + L_2 L_4) \]
\[ P_1^{1,2,3} = G_1 G_2 G_3 G_4 \quad \Delta_1 = 1 - (L_3 + L_4) \]
\[ P_2^{4,5,6} = G_5 G_6 G_7 G_8 \quad \Delta_2 = 1 - (L_1 + L_2) \]
\[ \frac{Y(s)}{R(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} \]
\[ \frac{x_5(s)}{R(s)} = ? \]
\[ \frac{Y(s)}{x_5(s)} = ? \]
Example 2.11

[Diagram of a control system with nodes and arrows indicating flow of variables]
\[ R(s) \xrightarrow{1} G_1 \xrightarrow{G_2} G_3 \xrightarrow{G_4} G_5 \xrightarrow{G_6} G_7 \xrightarrow{G_8} Y(s) \]

\[ -H_1 \xleftarrow{} G_5 \xleftarrow{} G_6 \]

\[ -H_2 \xleftarrow{} G_3 \xleftarrow{} G_4 \]

\[ -H_3 \xleftarrow{} G_1 \xleftarrow{} G_2 \]
Home Work 8

\[ Y/R = ? \]
\[ X/R = ? \]
\[ Y/X = ? \]
Home Work 9

Y/R=?

R → 1 a b c d 1 → y

Diagram:
- f
- e
- h
- i
- j
- m
- k
Home Work 10

Y/R=?
The Simulation of System

Assuming that a mode and the simulation are reliably accurate, computer simulation has the following advantages:

1. System performance can be observed under all conceivable conditions.
2. Results of field-system performance can be extrapolated with a simulation model for prediction purposes.
3. Decisions concerning future system presently in a conceptual stage can be examined.
4. Trials of system under test can be accomplished in a much-reduced period of time.
5. Simulation results can be obtained at lower cost than real experimentation.
6. Study of hypothetical situation can be achieved even when the hypothetical situation would be unrealizable in actual life at the present time.
7. Computer modeling and simulation is often the only feasible or safe technique to analyze and evaluate a system.
FIGURE 2.34 Analysis and design using a system model.
Example 2.12 Electric traction motor control

\[
\frac{\omega(s)}{v_{in}(s)} = ?
\]

FIGURE 2.35 Speed control of an electric traction motor.
FIGURE 2.35  Speed control of an electric traction motor.
The simulation of system using Matlab

\[ M\ddot{y} + b\dot{y} + ky = r(t) \]

The unforced dynamic response, \( y(t) \), of the spring-mass-damper mechanical system is

\[ y(t) = \frac{y(0)}{\sqrt{1 - \xi^2}} e^{-\xi\omega_n t} \sin(\omega_n \sqrt{1 - \xi^2} t + \theta) \]

Take Laplace and set I.C.=0

\[ [Ms^2 + bs + k]Y(s) = R(s) \]

\[ \frac{Y(s)}{R(s)} = \frac{1}{Ms^2 + bs + k} \]

Simulation with Simulink
FIGURE 2.40  Script to analyze the spring-mass-damper with MATLAB.
FIGURE 2.41  Spring-mass-damper unforced response.
Exercises

E2.4 E2.5 E2.8 E2.9 E2.18 E2.19 E2.20
P2.6 P2.7 P2.13, P2.17 P2.18 P2.25 P2.26 P2.34 P2.35 P2.45 P2.49