Ch3. Wave Properties of Particles

In a scanning electron microscope, an electron beam that scans a specimen causes secondary electrons to be ejected in numbers that vary with the angle of the surface. A suitable data display suggests the three-dimensional form of the specimen. The high resolution of this image of a red spider mite on a leaf is a consequence of the wave nature of moving electrons.
Introduction

1905: Einstein discovered "particle properties of waves".

1924: Louis de Broglie proposed that particles might show wave behavior.

De Broglie Wave

For a photon

\[ E = pc \rightarrow hf = pc \rightarrow p = \frac{hf}{c} = \frac{h}{\lambda} \]  

or \[ \lambda = \frac{h}{p} = \frac{h}{km_0 v} \]  

where \( k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \)

De Broglie suggested that (1) is a completely general one that applies to material particles as well as to photon.

The wave and particle aspects of moving bodies never be observed at the same time.
Which set of properties is most conspicuous depends on how its de Broglie wavelength compares with its dimensions and the dimensions of whatever it interacts with.

Example:
Find the de Broglie wavelengths of (a) a 46-g golf ball with a velocity of 30 m/s, and (b) an electron with a velocity of $10^7$ m/s.

Sol:

(a) $m_0 = 46 \times 10^{-3}$, $v = 30 \ll c$, so the relativistic correction can be neglected.

$$\lambda = \frac{h}{p} = \frac{h}{m_0 v} = \frac{6.626 \times 10^{-34}}{(46 \times 10^{-3}) \times 30} = 4.8 \times 10^{-34} \text{ m}.$$

(b) $m_0 = 9.11 \times 10^{-31}$, $v = 10^7 \ll c$, so the relativistic correction can be neglected.

$$\lambda = \frac{h}{p} = \frac{h}{m_0 v} = \frac{6.626 \times 10^{-34}}{(9.11 \times 10^{-31}) \times (10^7)} = 7.3 \times 10^{-11} \text{ m}.$$
The dimensions of atoms are comparable with this figure – the radius of hydrogen atom, for instant, is $5.3 \times 10^{-11}$ m. It is not surprising that the wave character of moving electron is the key to understanding atomic structure and behavior.

**Example**

Find the kinetic energy of a proton whose de Broglie wavelength is $1.000 \text{fm} = 1.000 \times 10^{-15}$ m, which is roughly the proton diameter.

**Sol:**

\[
\begin{align*}
\lambda &= 10^{-15} \text{ m}, \quad \rho = \frac{\hbar}{\lambda} = \frac{6.626 \times 10^{-34}}{10^{-15}} = 6.626 \times 10^{-19} \\
\rho C &= \rho \times (3 \times 10^8) = \frac{6.626 \times 10^{-19} \times (3 \times 10^8)}{1.6 \times 10^{-19}} = 1.24 \times 10^9 \text{ eV} \\
E &= \sqrt{\frac{1}{2} m v^2 + m v^2} = \sqrt{(0.938 \text{ GeV})^2 + (1.555 \text{ GeV})^2} = 1.555 \text{ GeV} \\
\therefore KE &= E - E_0 = (1.555 \text{ GeV}) - (0.938 \text{ GeV}) = 617 \text{ MeV}
\end{align*}
\]
de Broglie wave was verified by experiments involving the diffraction of electron by crystal. (We will talk about it later)

Wave of what? wave of probability
Fig. 5-19 Growth of two-slit interference pattern. The photo (d) is an actual two-slit electron interference pattern in which the film was exposed to millions of electrons. The pattern is identical to that usually obtained with photons. If the film were to be observed at various stages, such as after being struck by 28 electrons, then after about 1000 electrons, and again after about 10,000 electrons, the patterns of individually exposed grains would be similar to those shown in (a), (b), and (c), except that the exposed dots would be smaller than the dots drawn here. Note that there are no dots in the region of the interference minima. The probability of any point of the film being exposed is determined by wave theory, whether the film is exposed by electrons or photons. [Parts (a), (b), and (c) from E. R. Huggins, Physics 1, © by W. A. Benjamin, Inc., Menlo Park, California. Photo (d) courtesy of C. Jonsson.]
1. 我们用 $\Psi = \Psi(x, y, z, t)$ 来代表 matter wave 的大小。We call $\Psi$ the wave function. The value of the wave function associated with a moving body at a particular point $x, y, z$ in space at the time $t$ is related to the likelihood of finding the body there at the time.

2. The wave function $\Psi$ itself, however has no direct physical significance. The probability of experimentally finding the body described by the wave function $\Psi$ at the point $x, y, z$ at time $t$ is proportional to the value of $|\Psi|^2$ there at $t$. This interpretation was first made by Max Born in 1926.

3. Bragg wrote: Everything in the future is a wave, everything in the past is a particle.

解释 $|\Psi|^2$
Wave formula

Figure 10.1 Plot of $E(z, t) = A \sin(\omega t - \beta z)$ (a) with constant $t$, (b) with constant $z$.  

Figure 10.2 Plot of $E(z, t) = A \sin(\omega t - \beta z)$ at time (a) $t = 0$, (b) $t = T/4$, (c) $t = T/2$. $P$ moves along $+z$ direction with velocity $u$.  

分析
推導 $v_p$: phase velocity

由 $wt - kx + \phi = \text{常數}$

$\Rightarrow \frac{\partial}{\partial t}[wt - kx + \phi] = 0$

$\Rightarrow \omega - k\frac{dx}{dt} = 0 \Rightarrow \frac{dx}{dt} = \frac{\omega}{k} = v_p$

$k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f \quad \therefore \frac{\omega}{k} = \lambda f$
\[ \nu_p = \lambda f = \left( \frac{h}{p} \right) \left( \frac{E}{h} \right) = \left( \frac{h}{mv_g} \right) \left( \frac{mc^2}{h} \right) = \frac{c^2}{v_g} > c \]

\( v_g \)才是物體真正移動的速度，稱為 group velocity.

**Phase and Group velocity**

The wave representation of a moving body corresponds to a wave packet or wave group.

![Figure 6.9](image)

**FIGURE 6.9**

A localized wave packet, which is nonzero in an interval \( \pm \Delta x \) and zero elsewhere. A particle with this wave function would have a vanishingly small probability to be found outside the interval \( \pm \Delta x \).
Wave packet是如何形成的？我們以sound beat為例子來說明。

以聲音為例，若有兩個單音源之$f_1=440Hz$與$f_2=442Hz$，則此兩音源合成後we will hear a fluctuating sound of frequency 441Hz with two loudness peaks, call beats, per second.

分析
1. above represents a wave of angular frequency $w$ and wave number $k$.
that superimpose upon it a modulation of angular frequency $\frac{1}{2} \Delta w$ and

of wave number $\frac{1}{2} \Delta k$.

2. 如果有無窮多個 cosine wave with indefinitely small $\Delta k$ and $\Delta w$
difference, 則會形成一個完整的 group wave (or localized particle).
The de Broglie wave group associated with a moving body travels with the same velocity as the body.

Example

An electron has a de Broglie wavelength of $2.00 \times 10^{-12}$ m. Find its kinetic energy and the phase and group velocities of its de Broglie waves.

**Sol:**
Particle Diffraction

In 1927 Davison and Germer confirmed de Broglie’s hypothesis by demonstrating that electron beams are diffracted when they are scattered by the regular atomic arrays of crystals.

\[
\lambda = 2 \times 10^{-12}, \quad p = \frac{\hbar}{\lambda} \Rightarrow pc = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34} \text{ J s}) \times (3 \times 10^8 \text{ m/s})}{2 \times 10^{-12} \times 2} = 6.2 \times 10^5 \text{ eV}
\]

\[
E_0 = m_0 c^2 = 511 \text{ keV}, \quad KE = E - E_0 = \sqrt{E_0^2 + p^2 c^2} - E_0.
\]

\[
= \sqrt{(511 \text{ keV})^2 + (6.2 \times 10^5 \text{ eV})^2} - 511 \text{ keV} = 803 \text{ keV} - 511 \text{ keV} = 292 \text{ keV}
\]

\[
E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} = \frac{E_0}{\sqrt{1 - v^2/c^2}}
\]

\[
\Rightarrow v = c \sqrt{1 - \frac{E_0^2}{E^2}} = c \sqrt{1 - \left(\frac{511 \text{ keV}}{803 \text{ keV}}\right)^2} = 0.77c = \frac{\sqrt{7}}{2}
\]

\[
\sqrt{p} = \frac{c^2}{\sqrt{7}} = \frac{c^2}{0.77c} = 1.30 \text{ c}
\]
古典力學認為散射的角度與入射電子的能量無關（就如同打壁球一般），且應為 smoothly 的變化。但實際上卻有令人意外的結果。
De Broglie’s hypothesis suggested that electron waves were being diffracted by the target much as x-rays are diffracted by planes in a crystal.
**FIGURE 4.5**
The atoms of any one crystal define many different sets of equally spaced parallel planes, each containing many atoms. Three such sets of planes are shown shaded.

**Figure 3.8** The diffraction of the de Broglie waves by the target is responsible for the results of Davisson and Germer.
分析

\[ \theta = 65^\circ \text{ (晶面交角)} \]; \quad d = 0.091 \text{ nm} = \text{ spacing between planes.} \]

由 \( n\lambda = 2d\sin\theta \Rightarrow (n=1) \times \lambda = 2 \times (0.091 \text{ nm}) \times \sin(65^\circ) \Rightarrow \lambda = 0.165 \text{ nm} \]

由 \( \rho = \sqrt{\frac{mE}{\hbar}} = \sqrt{\frac{m \times 3E}{\hbar}} = \sqrt{2 \times (9.11 \times 10^{-31}) \times (54 \times 1.6 \times 10^{-19})} = 4 \times 10^{-4} \]

\[ \Rightarrow \lambda = \frac{\hbar}{\rho} = \frac{6.66 \times 10^{-34}}{4 \times 10^{-4}} = 1.66 \times 10^{-10} \text{ m} = 0.166 \text{ nm} \]

註: 此處我們忽略了電子進入 target 所獲得的 work function.
The Davisson–Germer experiment thus directly verifies de Broglie hypothesis of the wave nature of moving bodies.

事实上，如 neutron, atom 等都证实有 diffraction 的现象。

**FIGURE 6.3**
Diffraction rings produced by diffraction of waves in polycrystalline metal samples with (a) X-rays, (b) electrons, (c) neutrons.

**FIGURE 6.4**
Two-slit interference patterns produced by light and electrons.

**Electron Microscopes**
Figure 3.5 Because the wavelengths of the fast electrons in an electron microscope are shorter than those of the light waves in an optical microscope, the electron microscope can produce sharp images at higher magnifications. The electron beam in an electron microscope is focused by magnetic fields.
Electron Microscopes

The wave nature of moving electrons is the basis of the electron microscope, the first of which was built in 1932. The resolving power of any optical instrument, which is limited by diffraction, is proportional to the wavelength of whatever is used to illuminate the specimen. In the case of a good microscope that uses visible light, the maximum useful magnification is approximately 500×; higher magnifications give larger images but do not reveal any more detail. Electrons, however, have wavelengths very much shorter than those of visible light and are more easily controlled by electric and magnetic fields because of their charge. X-rays also have short wavelengths, but it is not (yet?) possible to focus them adequately.

In an electron microscope, current-carrying coils produce magnetic fields that act as lenses to focus an electron beam on a specimen and then produce an enlarged image on a fluorescent screen or photographic plate (Fig. 3.5). To prevent the beam from being scattered and thus blurring the image, a thin specimen is used and the entire system is evacuated.

The technology of magnetic “lenses” does not permit the full theoretical resolution of electron waves to be realized in practice. For instance, 100-keV electrons have wavelengths of 0.0037 nm, but the actual resolution they can provide in an electron microscope may be only about 0.1 nm. However, this is still a great improvement on the ~200-nm resolution of an optical microscope, and magnifications of over 1,000,000× have been achieved with electron microscopes.
an optical microscope, the electron microscope can produce sharp images at higher magnifications. The electron beam in an electron microscope is focused by magnetic fields.

Electron micrograph showing bacteriophage viruses in an Escherichia coli bacterium. The bacterium is approximately 1 \( \mu \text{m} \) across.
**Particle in a Box: Why the energy of a trapped particle is quantized**

(We will discuss this topic in detail in Ch.5)

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假設 particle 的總能量為 $E$ (此處沒有位能的變化，所以 $E = KE$)。Ignore relativistic consideration. From a wave point of view, a particle trapped in a box is like a standing wave in a string stretched between the box’s wall.
要在 box 內形成 standing wave 的條件為:

分析

$$L = \frac{n\lambda}{2}, \quad n=1, 2, 3 \ldots \quad \therefore \lambda = \frac{2L}{n}$$

$$p = \frac{h}{\lambda} = \frac{h}{(2L/n)} = \frac{nh}{2L}$$

$$\therefore \quad E = \frac{p^2}{2m} = \left(\frac{nh}{2L}\right)^2 = \frac{n^2h^2}{8mL}$$

Each permitted energy is called an energy level, $n$ is called “quantum number”.

![Energy Levels Diagram](image)
討論:
Any particle confined to a certain region of space (even if the region does not have a well-defined boundary)皆有以下的特性:

1. A trapped particle cannot have an arbitrary energy, as a free particle can.

2. A trapped particle cannot have zero energy. 因為有 standing wave 存在則必有波長，有波長則必有動量與能量。

3. Quantization of energy is conspicuous only when m and L are also small. 如此 $E_n - E_{n-1} = \Delta E$ (與 particle 本身所擁有的能量相比)才會明顯。

Example
An electron is in a box 0.10nm, which is the order of magnitude of atomic dimensions. Find its permitted energies.

Sol:
Indeed, energy quantization is prominent in the case of an atomic electron.
Example

A 10-g marble is in a box 10cm across. Find it permitted energies.

Sol:

\[ m = 10^{-2}, \ L = 10^{-1}, \ \therefore \ E_n = \frac{n^2 \hbar^2}{8mL^2} = \frac{n^2 \times (6.63 \times 10^{-34})^2}{8 \times 10^{-2} \times (10^{-1})^2} = 5.5 \times 10^{-14} \ n^2 J \]

The permissible energy levels are so close together, then, there is no way to determined wherever the marble can take on only those energies or any energy whatever.

**Uncertainty Principle**: We cannot know the future because we cannot know the present

In 1927 Heisenberg 提出Uncertainty principle: It is impossible to know both the exact position and exact momentum of an object at the same time.
A narrow wave packet (small $\Delta x$) corresponds to a large spread of wavelengths (large $\Delta k$). A wide wave packet (large $\Delta x$) corresponds to a small spread of wavelengths (small $\Delta k$).
$$k = \frac{2\pi}{\lambda} = \text{wave number}, \quad \frac{dk}{d\lambda} = \frac{-2\pi}{\lambda^2} \quad \therefore |\Delta k| = \frac{2\pi}{\lambda^2} |\Delta \lambda|$$

$$p = \frac{h}{\lambda} = \frac{h}{\left(\frac{2\pi}{\lambda}\right)} = 2\pi h k \Rightarrow \Delta p = 2\pi h \Delta k$$

When $\Delta x$ is small, it needs many different cosine waves to be合成，合成之后的wave packet $\Delta x$ enlarge. $\Delta k$ or $\Delta p$ to enlarge, 反之亦然.}

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**Figure 3.12** (a) A narrow de Broglie wave group. The position of the particle can be precisely determined, but the wavelength (and hence the particle's momentum) cannot be established because there are not enough waves to measure accurately. (b) A wide wave group. Now the wavelength can be precisely determined but not the position of the particle.
事實上，這可以從 Fourier series or Fourier transform theory 來解釋。

**Fourier Series:**

\[
f(x) = \sum_{n=0}^{\infty} a_n \cos(nkx)
\]

**Fourier Transform**

\[
f(x) = \int_{0}^{\infty} g(k) \cos kxdk
\]

\[
g(k) = \frac{1}{2\pi} \int_{0}^{\infty} f(x) \cos kxdx
\]

\[g(x)\text{為}f(x)\text{之Inverse Fourier Transform or spectrum of } f(x)\]
A narrow de Broglie wave group thus means a well-defined position ($\Delta x$ smaller) but a poorly defined wavelength and a large uncertainty $\Delta p$ in
the momentum of the particle the group represents. A wide wave group means a more precise momentum but a less precise position.

**Gaussian Function**

When a set of measurement is made of some quantity $x$ in which the experiment errors are random, the result is often Gaussian distribution whose form is a bell-shaped curve.
例如隨意丟向 $+x$ 方向目標為 $x_0$ 之石頭的位置分佈

上圖之 $f(x)$ 尚未被 normalized 分析
1. Gaussian Function: \( f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-(x-x_0)^2/2\sigma^2} \)

2. \( \sigma = \text{standard deviation} = \sqrt{\frac{1}{N} \sum_{k=1}^{N} (x_k - x_0)^2} \)

3. \( x_0 = \text{mean} \)

4. \( f(x) \) is normalized

\[ \Rightarrow \int_{-\infty}^{\infty} f(x) \, dx = 1. \]
\[ P_{x_1 \leq x \leq x_2} = \int_{x_1}^{x_2} f(x) \, dx \]

The probability that a measurement lies inside a certain range between \( x_1 \) and \( x_2 \).

5. The width of a Gaussian curve at half its maximum is \( 2.356 = 2\Delta x \)

\[ \Rightarrow f(\Delta x) = \frac{1}{\sigma \sqrt{2\pi}} \]

\[ \Rightarrow f(x = x_0 + \Delta x) = \frac{1}{2} f_{\text{max}} = \frac{1}{2} \left( \frac{1}{\sigma \sqrt{2\pi}} \right) \]

且 \( P_{x_0 \pm \Delta x} = \int_{x_0-\Delta x}^{x_0+\Delta x} f(x) \, dx = 0.683 = 68.3\% \)

6. \( P_{x_0 \pm 2\Delta x} = 95.4\% \)

7. \( f(x) \) 经过 Fourier transform 之后还是 Gaussian, 反之亦然

\[ \Rightarrow f(x) \xrightarrow{\text{Fourier}} g(k) \]

\[ \Rightarrow f(x) \xrightarrow{\text{Inverse Fourier}} \Delta k \]

8. 经过极限证明：在所有的概率函数中，当函数为 Gaussian 时，其 \( \Delta x \cdot \Delta k \) 为最小，且 \( \Delta x \cdot \Delta k = \frac{1}{2} \).

所以对于任意函数有：

\[ f(x) \approx \Delta x \cdot \Delta k \geq \frac{1}{2} \]
當$\Delta x$愈小時，$\Delta k$就愈大，反之亦然。

We cannot know exactly both where a particle is right now and what its momentum is. We cannot know the future for sure because we cannot know the present for sure. 這就是 de Broglie matter 之精神。一切都是“機率”的問題與結果。

The narrower the original wave packet – that is, the more precisely we know its position at that time – the more it spreads out because it is made up of a greater span of waves with different phase velocities (即$\Delta x\Delta k \geq \frac{1}{2}$)
Example

A measurement establishes the position of a proton with an accuracy of ±1.00×10⁻¹¹ m. Find the uncertainty in the proton’s 1.00s later. Assume \( v \ll c \).

Sol:

\[
\begin{align*}
\Delta x \text{ at } t=0, & \quad \frac{\Delta p}{\Delta x} \approx \frac{h}{2m} \quad \Rightarrow \quad \Delta p \approx \frac{\hbar}{2\Delta x} = \frac{1.054 \times 10^{-34}}{2 \times (10^{-11})} = 5.27 \times 10^{-24} \\
\therefore \Delta v & = \frac{\Delta p}{m} = \frac{5.27 \times 10^{-24}}{(1.672 \times 10^{-27})} = 3152 \text{ m/s} \\
\therefore 1 \text{ sec later: } & \Delta x \approx 1 \times \Delta v = 1 \times 3152 = 3.152 \text{ km}.
\end{align*}
\]
本例題的結果即顯示隨著時間的演進，$\Delta x$ 將會愈來愈大（在 $\Delta p$ 固定的情況下）。

以螞蟻被關在 match box 與 house 來作比喻：
Werner Heisenberg (1901–1976) was born in Duisberg, Germany, and studied theoretical physics at Munich, where he also became an enthusiastic skier and mountaineer. At Göttingen in 1924 as an assistant to Max Born, Heisenberg became uneasy about mechanical models of the atom: “Any picture of the atom that our imagination is able to invent is for that very reason defective,” he later remarked. Instead he conceived an abstract approach using matrix algebra. In 1925, together with Born and Pascual Jordan, Heisenberg developed this approach into a consistent theory of quantum mechanics, but it was so difficult to understand and apply that it had very little impact on physics at the time. Schrödinger's wave formulation of quantum mechanics the following year was much more successful; Schrödinger and others soon showed that the wave and matrix versions of quantum mechanics were mathematically equivalent.

In 1927, working at Bohr's institute in Copenhagen, Heisenberg developed a suggestion by Wolfgang Pauli into the uncertainty principle. Heisenberg initially felt that this principle was a consequence of the disturbances inevitably produced by any measuring process. Bohr, on the other hand, thought that the basic cause of the uncertainties was the wave-particle duality, so that they were built into the natural world rather than solely the result of measurement. After much argument Heisenberg came around to Bohr's view. (Einstein, always skeptical about quantum mechanics, said after a lecture by Heisenberg on the uncertainty principle: “Marvelous, what ideas the young people have these days. But I don't believe a word of it.”) Heisenberg received the Nobel Prize in 1932.

Heisenberg was one of the very few distinguished scientists to remain in Germany during the Nazi period. In World War II he led research there on atomic weapons, but little progress had been made by the war's end. Exactly why remains unclear, although there is no evidence that Heisenberg, as he later claimed, had moral qualms about creating such weapons and more or less deliberately dragged his feet. Heisenberg recognized early that "an explosive of unimaginable consequences" could be developed, and he and his group should have been able to have gotten farther than they did. In fact, alarmed by the news that Heisenberg was working on an atomic bomb, the U.S. government sent the former Boston Red Sox catcher Moe Berg to shoot Heisenberg during a lecture in neutral Switzerland in 1944. Berg, sitting in the second row, found himself uncertain from Heisenberg's remarks about how advanced the German program was, and kept his gun in his pocket.
\[ \Delta x(t = 0) \] 愈小,則一段時間後,其 \( \Delta x \) 的誤差百分比也愈大(因 \( \Delta x(t = 0) \) 愈小,則 \( \Delta p(t = 0) \) 就愈大。

Uncertainty Principle 的進一步思考:

1. 事實上我們永遠都無法百分之百確知 particle 目前的位置,原因是任何的量測都會引入干擾而產生誤差。

2. 要能百分之百的確定 particle 的位置,最保險的辦法就是將它“鎖住”。但是這個動作確完全的破壞了它原本的動量。

3. If we are to see the electron 來量測電子的位置,若使用光源的 \( \lambda \) 愈小,則愈清楚,所以電子的 \( \Delta x \) 就愈小,但由於光子干擾電子的動量太大,反而使得電子的 \( \Delta p \) 變大。反之亦然。
Applying the Uncertainty Principle: A useful tool, not just a negative statement.

Example

A typical atomic nucleus is about $5.0 \times 10^{-15}$ m in radius. Use the uncertainty principle to place a lower limit on the energy an electron must have if it is to be part of a nucleus.

Sol:

\[
\Delta p \geq \frac{\hbar}{2\Delta x} = \frac{1.054 \times 10^{-34}}{2 \times (5 \times 10^{-15})} = 1.05 \times 10^{-20}
\]

Assuming $\Delta p \approx \Delta p_e$, then for an electron with $KE = \frac{1}{2} m_e c^2 - E_0$

\[
E_0 = \frac{p^2 c^2}{2} = (1.1 \times 10^{-20}) \times (3 \times 10^8) = 20 \text{ MeV}
\]

事實上我們從來就沒發現過有如此超級動能的電子從原子核跑出來, 據此我們可以肯定電子不可能待在原子核內。
Example

A hydrogen atom is $5.3 \times 10^{-11}$ m in radius. Use the uncertainty principle to estimate the minimum energy an electron can have in this atom.

Sol:

$$\Delta x = 5.3 \times 10^{-11}, \quad \Delta p = \frac{\hbar}{2\Delta x} = 9.9 \times 10^{-25} \approx 1$$

$$\Rightarrow KE \approx \frac{p^2}{2m} = \frac{9.9 \times 10^{-25}}{2 \times 9.11 \times 10^{-31}} = 3.4 \text{eV}$$

Actually, the kinetic energy of an electron in the lowest energy level of a hydrogen atom is 13.6eV.

Energy & Time

Uncertainty principle 的另一個等效的表示式為:

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$
An “excited” atom gives up its energy by emitting a photon of characteristic frequency. The average period that elapse between the excitation of an atom and the time it radiates is 1.0x10⁻⁸ s. Find the inherent uncertainty in the frequency of the photon.

\[ \Delta \omega = \frac{\hbar}{\Delta t} \Rightarrow \Delta \omega \approx \frac{\hbar}{1.0 \times 10^{-8}} \Rightarrow \Delta f \approx \frac{f}{\Delta t} \approx \frac{1}{4 \pi \times 10^{-8}} \approx 8 \times 10^6 \text{ Hz} \]

\[ \Delta f / f = 1.6 \times 10^{-8} \]

So if we measure the photon’s frequency in a time of 1.0x10⁻⁸, the error in the frequency will be so small (8x10⁶ Hz) that it’s negligible. This is illustrated in the example below where we measure the frequency of a photon whose frequency is, say, 5.0x10¹⁴ Hz.