1. (a) \( \lim_{x \to a} \frac{f(x)}{g(x)} \) is an indeterminate form of type \( \frac{0}{0} \).

(b) \( \lim_{x \to a} \frac{f(x)}{g(x)} = 0 \) because the numerator approaches 0 while the denominator becomes large.

(c) \( \lim_{x \to a} \frac{h(x)}{p(x)} = 0 \) because the numerator approaches a finite number while the denominator becomes large.

(d) If \( \lim_{x \to a} p(x) = \infty \) and \( f(x) \to 0 \) through positive values, then \( \lim_{x \to a} \frac{p(x)}{f(x)} = \infty \). [For example, take \( a = 0 \), \( p(x) = \frac{1}{x^2} \), and \( f(x) = x^2 \).] If \( f(x) \to 0 \) through negative values, then \( \lim_{x \to a} \frac{p(x)}{f(x)} = -\infty \). [For example, take \( a = 0 \), \( p(x) = \frac{1}{x^2} \), and \( f(x) = -x^2 \).] If \( f(x) \to 0 \) through both positive and negative values, then the limit might not exist. [For example, take \( a = 0 \), \( p(x) = \frac{1}{x^2} \), and \( f(x) = x \).]

(e) \( \lim_{x \to a} \frac{p(x)}{q(x)} \) is an indeterminate form of type \( \frac{\infty}{\infty} \).

2. (a) \( \lim_{x \to a} [f(x)p(x)] \) is an indeterminate form of type \( 0 \cdot \infty \).

(b) When \( x \) is near \( a \), \( p(x) \) is large and \( h(x) \) is near 1, so \( h(x)p(x) \) is large. Thus, \( \lim_{x \to a} [h(x)p(x)] = \infty \).

(c) When \( x \) is near \( a \), \( p(x) \) and \( q(x) \) are both large, so \( p(x)q(x) \) is large. Thus, \( \lim_{x \to a} [p(x)q(x)] = \infty \).

3. (a) When \( x \) is near \( a \), \( f(x) \) is near 0 and \( p(x) \) is large, so \( f(x) - p(x) \) is large negative. Thus, \( \lim_{x \to a} [f(x) - p(x)] = -\infty \).

(b) \( \lim_{x \to a} [p(x) - q(x)] \) is an indeterminate form of type \( \infty - \infty \).

(c) When \( x \) is near \( a \), \( p(x) \) and \( q(x) \) are both large, so \( p(x) + q(x) \) is large. Thus, \( \lim_{x \to a} [p(x) + q(x)] = \infty \).

4. (a) \( \lim_{x \to a} [f(x)]^{g(x)} \) is an indeterminate form of type \( 0^0 \).

(b) If \( y = [f(x)]^{g(x)} \), then \( \ln y = p(x) \ln f(x) \). When \( x \) is near \( a \), \( p(x) \to \infty \) and \( \ln f(x) \to -\infty \), so \( \ln y \to -\infty \).

Therefore, \( \lim_{x \to a} [f(x)]^{g(x)} = \lim_{x \to a} y = \lim_{x \to a} e^{\ln y} = 0 \), provided \( f^p \) is defined.

(c) \( \lim_{x \to a} [h(x)]^{g(x)} \) is an indeterminate form of type \( 1^\infty \).

(d) \( \lim_{x \to a} [p(x)]^{f(x)} \) is an indeterminate form of type \( \infty^0 \).

(e) If \( y = [p(x)]^{g(x)} \), then \( \ln y = q(x) \ln p(x) \). When \( x \) is near \( a \), \( q(x) \to \infty \) and \( \ln p(x) \to \infty \), so \( \ln y \to \infty \).

Therefore, \( \lim_{x \to a} [p(x)]^{g(x)} = \lim_{x \to a} y = \lim_{x \to a} e^{\ln y} = \infty \).

(f) \( \lim_{x \to a} [p(x)]^{g(x)} = \lim_{x \to a} [p(x)]^{1/q(x)} \) is an indeterminate form of type \( \infty^0 \).
7. This limit has the form \( \frac{0}{0} \):
\[
\lim_{x \to 1} \frac{x^5 - 1}{x^5 - 1} = \lim_{x \to 1} \frac{9x^5}{5x^4} = \frac{9}{5} \lim_{x \to 1} x^4 = \frac{9}{5} (1) = \frac{9}{5}
\]

9. This limit has the form \( \frac{0}{0} \):
\[
\lim_{x \to (\pi/2)^+} \frac{\cos x}{1 - \sin x} = \lim_{x \to (\pi/2)^+} \frac{\sin x}{-\cos x} = \lim_{x \to (\pi/2)^+} \tan x = -\infty.
\]

17. \( \lim_{x \to 0^+} \frac{\ln x}{x} = -\infty \) since \( \ln x \to -\infty \) as \( x \to 0^+ \) and dividing by small values of \( x \) just increases the magnitude of the quotient \( (\ln x)/x \). L’Hospital’s Rule does not apply.

19. This limit has the form \( \frac{\infty}{\infty} \):
\[
\lim_{x \to \infty} \frac{e^x}{x^3} = \lim_{x \to \infty} \frac{e^x}{3x^2} = \lim_{x \to \infty} \frac{e^x}{6x} = \lim_{x \to \infty} \frac{e^x}{6} = \infty
\]

21. This limit has the form \( \frac{0}{0} \):
\[
\lim_{x \to 0} \frac{e^x - 1 - x}{x^2} = \lim_{x \to 0} \frac{e^x - 1}{2x} = \lim_{x \to 0} \frac{e^x}{2} = \frac{1}{2}
\]

27. This limit has the form \( \frac{0}{0} \):
\[
\lim_{x \to 0} \frac{\sin^{-1} x}{x} = \lim_{x \to 0} \frac{1/\sqrt{1-x^2}}{1} = \lim_{x \to 0} \frac{1}{\sqrt{1-x^2}} = \frac{1}{1} = 1
\]

29. This limit has the form \( \frac{0}{0} \):
\[
\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \lim_{x \to 0} \frac{\sin x}{2x} = \lim_{x \to 0} \frac{\cos x}{2} = \frac{1}{2}
\]

31. \( \lim_{x \to 0} x + \sin x = 0 + 0 = 0 = 0 \). L’Hospital’s Rule does not apply.

33. This limit has the form \( \frac{0}{x} \). We’ll change it to the form \( \frac{\infty}{\infty} \):
\[
\lim_{x \to 1} \frac{x - \ln x}{1 + \cos \pi x} = \lim_{x \to 1} \frac{-1 + 1/x}{-\pi \sin \pi x} = \lim_{x \to 1} \frac{-1/x^2}{-\pi^2 \cos \pi x} = \frac{-1}{-\pi^2 (-1)} = -\frac{1}{\pi^2}
\]

41. This limit has the form \( \infty \cdot 0 \). We’ll change it to the form \( \frac{\infty}{\infty} \):
\[
\lim_{x \to 0} \cot 2x \sin 6x = \lim_{x \to 0} \frac{\sin 6x}{\tan 2x} = \lim_{x \to 0} \frac{6 \cos 6x}{2 \sec^2 2x} = \frac{6(1)}{2(1)^2} = 3
\]

43. This limit has the form \( \infty \cdot 0 \):
\[
\lim_{x \to -\infty} \frac{x^3 e^{-x^2}}{2e^{x^2}} = \lim_{x \to -\infty} \frac{3x^2}{2e^{x^2}} = \lim_{x \to -\infty} \frac{3x}{4xe^{x^2}} = 0
\]

45. This limit has the form \( 0 \cdot (-\infty) \):
\[
\lim_{x \to 1^+} \ln x \tan(\pi x/2) = \lim_{x \to 1^+} \frac{\ln x}{\cot(\pi x/2)} = \lim_{x \to 1^+} \frac{1/x}{(-\pi/2) \cot(\pi x/2)} = \frac{1}{(-\pi/2)(1)^2} = -\frac{2}{\pi}
\]
49. We will multiply and divide by the conjugate of the expression to change the form of the expression.

\[
\lim_{x \to \infty} \left( \frac{\sqrt{x^2 + x} - x}{1} \right) = \lim_{x \to \infty} \frac{\sqrt{x^2 + x} + x}{\sqrt{x^2 + x} + x} \cdot \frac{\sqrt{x^2 + x} - x}{x} = \lim_{x \to \infty} \left( \frac{x^2 + x}{\sqrt{x^2 + x} + x} \right) = \lim_{x \to \infty} \frac{1}{\sqrt{1 + 1/x} + 1} = \frac{1}{2}
\]

As an alternate solution, write \( \sqrt{x^2 + x} - x \) as \( \sqrt{x^2 + x} - \sqrt{x^2} \), factor out \( \sqrt{x^2} \), rewrite as \( (\sqrt{1+1/x} - 1)/(1/x) \), and apply l'Hospital's Rule.

55. \( y = (1 - 2x)^{1/x} \Rightarrow \ln y = \frac{1}{x} \ln(1 - 2x) \), so \( \lim_{x \to 0} \ln y = \lim_{x \to 0} \frac{\ln(1 - 2x)}{x} = \lim_{x \to 0} \frac{-2(1 - 2x)}{1} = -2 \Rightarrow \)

\[
\lim_{x \to 0} (1 - 2x)^{1/x} = \lim_{x \to 0} e^{\ln y} = e^{-2}.
\]

59. \( y = x^{1/x} \Rightarrow \ln y = \left( \frac{1}{x} \right) \ln x \Rightarrow \lim_{x \to \infty} \ln y = \lim_{x \to \infty} \frac{1}{x^2} = \lim_{x \to \infty} \frac{1}{x^2} = 0 \Rightarrow \)

\[
\lim_{x \to \infty} x^{1/x} = \lim_{x \to \infty} e^{\ln y} = e^0 = 1
\]

63. \( y = (\cos x)^{1/x^2} \Rightarrow \ln y = \frac{1}{x^2} \ln \cos x \Rightarrow \lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} \frac{\ln \cos x}{x^2} = \lim_{x \to 0^+} \frac{-\tan x}{2x^2} = \lim_{x \to 0^+} \frac{-\sec^2 x}{2} = \frac{1}{2}
\]

\[
\lim_{x \to 0^+} e^{\ln y} = e^{-1/2} = 1/\sqrt{e}
\]

66. \[
\lim_{x \to \infty} e^x = \lim_{n \to \infty} \frac{1}{e^{-n}} = \lim_{n \to \infty} \frac{e^n}{n!} = \infty
\]

70. This limit has the form \( \frac{\infty}{\infty} \) since \( \lim_{x \to \infty} \frac{\ln x}{x/p^p - 1} = \lim_{x \to \infty} \frac{1}{p^p} = 0 \) since \( p > 0 \).

78. Let the radius of the circle be \( r \). We see that \( A(\theta) \) is the area of the whole figure (a sector of the circle with radius 1), minus the area of \( \triangle OPR \). But the area of the sector of the circle is \( \frac{1}{2}r^2\theta \) (see Reference Page 1), and the area of the triangle is \( \frac{1}{2}|PQ| = \frac{1}{2}r \sin \theta = \frac{1}{2}r^2 \sin \theta \). So we have \( A(\theta) = \frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin \theta = \frac{1}{2}r^2(\theta - \sin \theta) \). Now by elementary trigonometry, \( B(\theta) = \frac{1}{2}(QR||PQ| = \frac{1}{2}(r - |OQ|)|PQ| = \frac{1}{2}(r - r \cos \theta)(r \sin \theta) = \frac{1}{2}r^2(1 - \cos \theta) \sin \theta \).

So the limit we want is

\[
\lim_{\theta \to 0^+} \frac{A(\theta)}{B(\theta)} = \lim_{\theta \to 0^+} \frac{\frac{1}{2}r^2(\theta - \sin \theta)}{\frac{1}{2}r^2(1 - \cos \theta) \sin \theta + \sin \theta (\sin \theta)} = \lim_{\theta \to 0^+} \frac{1 - \cos \theta}{\cos \theta - \cos^2 \theta + \sin^2 \theta} = \lim_{\theta \to 0^+} \frac{\sin \theta}{-\sin \theta - 2 \cos \theta (-\sin \theta) + 2 \sin \theta (\cos \theta)} = \lim_{\theta \to 0^+} \frac{1}{1 + 4 \cos \theta} = \frac{1}{3}
\]
79. Since \( f(2) = 0 \), the given limit has the form \( \frac{0}{0} \):

\[
\lim_{x \to 0} \frac{f(2 + 3x) + f(2 + 5x)}{x} = \lim_{x \to 0} \frac{f'(2 + 3x) \cdot 3 + f'(2 + 5x) \cdot 5}{1} - f'(2) \cdot 3 + f'(2) \cdot 5 = 8f'(2) = 8 \cdot 7 = 56
\]

80. \( L = \lim_{x \to 0} \left( \frac{\sin 2x}{x^2} + \frac{b}{x^2} \right) = \lim_{x \to 0} \frac{\sin 2x + ax^2 + bx}{x^2} \lim_{x \to 0} 2 \cos 2x + 3ax^2 + b \) As \( x \to 0 \), \( 3x^2 \to 0 \), and

\( 2 \cos 2x + 3ax^2 + b \to b + 2 \), so the last limit exists only if \( b + 2 = 0 \), that is, \( b = -2 \). Thus,

\[
\lim_{x \to 0} \frac{2 \cos 2x + 3ax^2 - 2}{3x^2} = \lim_{x \to 0} \frac{-4 \sin 2x + 6ax}{6x} = \lim_{x \to 0} \frac{-8 \cos 2x + 6a}{6} = \frac{6a - 8}{6}, \text{ which is equal to 0 if and only if } a = \frac{4}{3}. \text{ Hence, } L = 0 \text{ if and only if } b = -2 \text{ and } a = \frac{4}{3}.
\]

81. Since \( \lim_{h \to 0} [f(x + h) - f(x - h)] = f(x) - f(x) = 0 \) (\( f \) is differentiable and hence continuous) and \( \lim_{h \to 0} 2h = 0 \), we use L'Hospital's Rule:

\[
\lim_{h \to 0} \frac{f(x + h) - f(x - h)}{2h} = \lim_{h \to 0} \frac{f(x + h)(1) - f(x - h)(-1)}{2} = \frac{f'(x) + f'(x)}{2} = \frac{2f'(x)}{2} = f'(x)
\]

\( \frac{f(x + h) - f(x - h)}{2h} \) is the slope of the secant line between \( (x - h, f(x - h)) \) and \( (x + h, f(x + h)) \). As \( h \to 0 \), this line gets closer to the tangent line and its slope approaches \( f'(x) \).