2. (a) (i) \( L_6 = \sum_{i=1}^{6} f(x_{i-1}) \Delta x \quad [\Delta x = \frac{12-0}{6} = 2] \)

\[ = 2[f(0) + f(2) + f(4) + f(6) + f(8) + f(10)] \]
\[ \approx 2(9.8 + 8.2 + 7.3 + 5.9 + 4.1) \]
\[ = 2(43.3) = 86.6 \]

(ii) \( R_6 = L_6 + 2 \cdot f(12) - 2 \cdot f(0) \)
\[ \approx 86.6 + 2(1) - 2(9) = 70.6 \]

[Add area of rightmost lower rectangle
and subtract area of leftmost upper rectangle.]

(iii) \( M_6 = \sum_{i=1}^{6} f(x_i^*) \Delta x \)

\[ = 2[f(1) + f(3) + f(5) + f(7) + f(9) + f(11)] \]
\[ \approx 2(8.9 + 8.5 + 7.8 + 6.6 + 5.1 + 2.8) \]
\[ = 2(39.7) = 79.4 \]

(b) Since \( f \) is decreasing, we obtain an overestimate by using left endpoints; that is, \( L_6 \).

(c) Since \( f \) is decreasing, we obtain an underestimate by using right endpoints; that is, \( R_6 \).

(d) \( M_6 \) gives the best estimate, since the area of each rectangle appears to be closer to the true area than the overestimates and underestimates in \( L_6 \) and \( R_6 \).
5. (a) \( f(x) = 1 + x^2 \) and \( \Delta x = \frac{2 - (-1)}{3} = 1 \) \( \Rightarrow \)

\[
R_2 = 1 \cdot f(0) + 1 \cdot f(1) + 1 \cdot f(2) = 1 \cdot 1 + 1 \cdot 2 + 1 \cdot 5 = 8.
\]

\[
\Delta x = \frac{2 - (-1)}{6} = 0.5 \ \Rightarrow
\]

\[
R_6 = 0.5[f(-0.5) + f(0.5) + f(1) + f(1.5) + f(2)]
= 0.5(1.25 + 1 + 1.25 + 2 + 3.25 + 5)
= 0.5(13.75) = 6.875
\]

(b) \( L_3 = 1 \cdot f(-1) + 1 \cdot f(0) + 1 \cdot f(1) = 1 \cdot 2 + 1 \cdot 1 + 1 \cdot 2 = 5 \)

\[
L_6 = 0.5[f(-1) + f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5)]
= 0.5(2 + 1.25 + 1 + 1.25 + 2 + 3.25)
= 0.5(10.75) = 5.375
\]

(c) \( M_3 = 1 \cdot f(-0.5) + 1 \cdot f(0.5) + 1 \cdot f(1.5) \)

\[
= 1 \cdot 1.25 + 1 \cdot 1.25 + 1 \cdot 3.25 = 5.75
\]

\[
M_6 = 0.5[f(-0.75) + f(-0.25) + f(0.25) + f(0.75) + f(1.25) + f(1.75)]
= 0.5(1.5625 + 1.0625 + 1.0625 + 1.5625 + 2.5625 + 4.0625)
= 0.5(11.875) = 5.9375
\]

(d) \( M_3 \) appears to be the best estimate.

17. \( f(x) = \sqrt{x}, \ 1 \leq x \leq 16. \quad \Delta x = (16 - 1)/n = 15/n \) and \( x_i = 1 + i \Delta x = 1 + 15i/n. \)

\[
A = \lim_{n \to \infty} R_n = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x = \lim_{n \to \infty} \sum_{i=1}^{n} \sqrt{1 + \frac{15i}{n}} \cdot \frac{15}{n}.
\]

19. \( f(x) = x \cos x, \ 0 \leq x \leq \frac{\pi}{2}. \quad \Delta x = (\frac{\pi}{2} - 0)/n = \frac{\pi}{2} \) \( n_i = 0 + i \Delta x = \frac{\pi}{2} i/n. \)

\[
A = \lim_{n \to \infty} R_n = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x = \lim_{n \to \infty} \sum_{i=1}^{n} \frac{i \pi}{2n} \cos \left( \frac{i \pi}{2n} \right) \cdot \frac{\pi}{2n}.
\]
20. \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{2}{n} \left( 5 + \frac{2i}{n} \right)^{10} \) can be interpreted as the area of the region lying under the graph of \( y = (5 + x)^{10} \) on the interval \([0, 2]\), since for \( y = (5 + x)^{10} \) on \([0, 2]\) with \( \Delta x = \frac{2 - 0}{n} = \frac{2}{n} \), \( x_i = 0 + i \Delta x = \frac{2i}{n} \), and \( x_i^* = x_i \), the expression for the area is \( A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x = \lim_{n \to \infty} \sum_{i=1}^{n} \left( 5 + \frac{2i}{n} \right)^{10} \cdot \frac{2}{n} \). Note that the answer is not unique. We could use \( y = x^{10} \) on \([5, 7]\) or, in general, \( y = ((5 - n) + x)^{10} \) on \([n, n + 2]\).

21. \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{\pi}{4n} \tan \frac{i \pi}{4n} \) can be interpreted as the area of the region lying under the graph of \( y = \tan x \) on the interval \([0, \frac{\pi}{4}]\), since for \( y = \tan x \) on \([0, \frac{\pi}{4}]\) with \( \Delta x = \frac{\pi/4 - 0}{n} = \frac{\pi}{4n} \), \( x_i = 0 + i \Delta x = \frac{i \pi}{4n} \), and \( x_i^* = x_i \), the expression for the area is \( A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x = \lim_{n \to \infty} \sum_{i=1}^{n} \tan \left( \frac{i \pi}{4n} \right) \frac{\pi}{4n} \). Note that this answer is not unique, since the expression for the area is the same for the function \( y = \tan(x - k\pi) \) on the interval \([k\pi, k\pi + \frac{\pi}{4}]\), where \( k \) is any integer.