1. Find the local maximum and minimum values and saddle point(s) of the function \( f(x, y) = x^4 + y^4 - 4xy + 2 \).
2. Figure 2 shows some level curves of a function \( f \).

(4%) (1) Use the figure to estimate the saddle points and the points where local minimum and maximum values occur.

saddle points: \((0, 0)\); local minimum: \((1, 1)\); local maximum: \(\text{None}\)

(2) Determine whether the discriminant \( D(0, 0) \) is positive or negative. \( D(0, 0): \) 

\[
\begin{align*}
\phi_x &= 4x^3 - 4y = 0 \\
\phi_y &= 4y^3 - 4x = 0 \\
\Rightarrow & \quad \phi_y = x^3 \\
\Rightarrow & \quad x = (x^\frac{1}{3})^3 \\
\Rightarrow & \quad x = 0, x = \pm 1
\end{align*}
\]

When \( x = 0 \), \( y = 0 \).
When \( x = 1 \), \( y = 1 \).
When \( x = -1 \), \( y = -1 \).

\((0, 0), (1, 1), (-1, 1)\) are critical points.

\(\phi_{xx} = 12x^2, \phi_{yy} = -4, \phi_{yx} = 12y^2\)

\[
D(x, y) = \begin{vmatrix} 12x^2 & -4 \\ -4 & 12y^2 \end{vmatrix} = 144x^2y^2 - 16
\]

(i) At \((0, 0)\): \( D(0, 0) = -16 < 0 \), \((0, 0)\) is a saddle point.
(ii) At \((1, 1)\): \( D(1, 1) > 0 \), and \( \phi_{xx}(1, 1) > 0 \),

\[ \phi_x(1, 1) = 0 \] is a local min. value of \( f \).
(iii) At \((-1, 1)\): \( D(-1, 1) > 0 \) and \( \phi_{xx}(-1, 1) > 0 \),

\[ \phi_x(-1, 1) = 0 \] is a local min. value of \( f \).

\( f \) has no local max. values.