Calculus Exercises (8.7)

1. Find the Maclaurin series for \( f(x) = e^{x^2} \) and \( g(x) = x^3 e^{x^2} \). Find the radius of convergence for both series.

2. Find the Taylor series for \( f(x) = \cos x \) centered at \( \pi \) and its radius of convergence.

3. Given that the Maclaurin series for \( \cos x \) is \[ \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \].
   (1) Use the first three terms of the series to approximate the integral \( \int_{0}^{1} \cos(x^2) \, dx \).
   (2) Find the sum of the series \( \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n} (2n)!} \).
   (3) Use the series to evaluate the limit \( \lim_{x \to 0} \frac{1 - \cos x}{x^2} \).

4. (1) Use the binomial series to find the Maclaurin series of \( f(x) = \frac{1}{\sqrt{1 + x^3}} \) and its radius of convergence.
   (2) Use the part (1) to evaluate \( f^{(9)}(0) \).