12.6 Cylinders (柱面) and Quadric Surfaces (二次曲面)

Cylinders

**Example** Sketch the graph of the surfaces (1) $z = y^2$ (2) $y^2 + x^2 = 1$.

[Sol]:

(1) Parabolic cylinder (拋物柱面)

(2) Circular cylinder (圓柱面)
Quadric Surfaces

Example 1  Sketch the graph of the surface $z = y^2 - x^2$.
$z = y^2 - x^2$

Hyperbolic paraboloid

(雙曲拋物面)
All traces are ellipses.

If $a = b = c$, the ellipsoid is a sphere.
Elliptic Paraboloid

\[ \frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2} \]

Horizontal traces are ellipses.

Vertical traces are parabolas.

The variable raised to the first power indicates the axis of the paraboloid.
Horizontal traces are hyperbolas.

Vertical traces are parabolas.

The case where $c < 0$ is illustrated.
Horizontal traces are ellipses.

Vertical traces in the planes $x = k$ and $y = k$ are hyperbolas if $k \neq 0$ but are pairs of lines if $k = 0$. 

\[
\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}
\]
Hyperboloid of One Sheet

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \]

Horizontal traces are ellipses.

Vertical traces are hyperbolas.

The axis of symmetry corresponds to the variable whose coefficient is negative.
Exercise:

Draw graphs of the quadric surfaces (a) \( x^2 + y^2 = z \) (b) \( \frac{x^2}{4} + y^2 - \frac{z^2}{2} = 1 \)
(a) $x^2 + y^2 = z$

(b) $\frac{x^2}{4} + y^2 - \frac{z^2}{2} = 1$