3. (a) \( f(x) \) approaches 2 as \( x \) approaches 1 from the left, so \( \lim_{x \to 1^-} f(x) = 2 \).

(b) \( f(x) \) approaches 3 as \( x \) approaches 1 from the right, so \( \lim_{x \to 1^+} f(x) = 3 \).

(c) \( \lim_{x \to 1} f(x) \) does not exist because the limits in part (a) and part (b) are not equal.

(d) \( f(x) \) approaches 4 as \( x \) approaches 5 from the left and from the right, so \( \lim_{x \to 5} f(x) = 4 \).

(e) \( f(5) \) is not defined, so it doesn’t exist.

4. (a) \( \lim_{x \to 0} f(x) = 3 \)  
(b) \( \lim_{x \to 3^-} f(x) = 4 \)  
(c) \( \lim_{x \to 3^+} f(x) = 2 \)  
(d) \( \lim_{x \to 3} f(x) \) does not exist because the limits in part (b) and part (c) are not equal.  
(e) \( f(3) = 3 \)

5. (a) \( \lim_{t \to 0^-} g(t) = -1 \)  
(b) \( \lim_{t \to 0^+} g(t) = -2 \)  
(c) \( \lim_{t \to 0} g(t) \) does not exist because the limits in part (a) and part (b) are not equal.  
(d) \( \lim_{t \to 2^-} g(t) = 2 \)  
(e) \( \lim_{t \to 2^+} g(t) = 0 \)  
(f) \( \lim_{t \to 2} g(t) \) does not exist because the limits in part (d) and part (e) are not equal.

(g) \( g(2) = 1 \)  
(h) \( \lim_{t \to 4} g(t) = 3 \)

6. \( \lim_{x \to a} f(x) \) exists for all \( a \) except \( a = \pm 1 \).
7. \[ \lim_{x \to 1^-} f(x) = 2, \quad \lim_{x \to 1^+} f(x) = -2, \quad f(1) = 2 \]

\[ \graph \]

9. \[ \lim_{x \to 3^+} f(x) = 4, \quad \lim_{x \to 3^-} f(x) = 2, \quad \lim_{x \to -2} f(x) = 2, \]
\[ f(3) = 3, \quad f(-2) = 1 \]

\[ \graph \]

11. For \( f(x) = \frac{x^2 - 2x}{x^2 - x - 2} \):

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>0.714286</td>
<td>1.9</td>
<td>0.655172</td>
</tr>
<tr>
<td>2.1</td>
<td>0.677419</td>
<td>1.95</td>
<td>0.661017</td>
</tr>
<tr>
<td>2.05</td>
<td>0.672131</td>
<td>1.99</td>
<td>0.665552</td>
</tr>
<tr>
<td>2.01</td>
<td>0.667774</td>
<td>1.995</td>
<td>0.666110</td>
</tr>
<tr>
<td>2.005</td>
<td>0.667221</td>
<td>1.999</td>
<td>0.666556</td>
</tr>
<tr>
<td>2.001</td>
<td>0.666778</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It appears that \[ \lim_{x \to 2} \frac{x^2 - 2x}{x^2 - x - 2} = 0.6 = \frac{2}{3} . \]
12. For \( f(x) = \frac{x^2 - 2x}{x^2 - x - 2} \):

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-0.5</td>
<td>-1</td>
</tr>
<tr>
<td>-0.9</td>
<td>-9</td>
</tr>
<tr>
<td>-0.95</td>
<td>-19</td>
</tr>
<tr>
<td>-0.99</td>
<td>-99</td>
</tr>
<tr>
<td>-0.999</td>
<td>-999</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>-1.5</td>
<td>3</td>
</tr>
<tr>
<td>-1.1</td>
<td>11</td>
</tr>
<tr>
<td>-1.01</td>
<td>101</td>
</tr>
<tr>
<td>-1.001</td>
<td>1001</td>
</tr>
</tbody>
</table>

It appears that \( \lim_{x \to -1} \frac{x^2 - 2x}{x^2 - x - 2} \) does not exist since

\[
f(x) \to \infty \text{ as } x \to -1^- \text{ and } f(x) \to -\infty \text{ as } x \to -1^+.
\]

13. For \( f(x) = \frac{\sin x}{x + \tan x} \):

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>±1</td>
<td>0.329033</td>
</tr>
<tr>
<td>±0.5</td>
<td>0.458209</td>
</tr>
<tr>
<td>±0.2</td>
<td>0.493331</td>
</tr>
<tr>
<td>±0.1</td>
<td>0.498333</td>
</tr>
<tr>
<td>±0.05</td>
<td>0.499583</td>
</tr>
<tr>
<td>±0.01</td>
<td>0.499983</td>
</tr>
</tbody>
</table>

It appears that \( \lim_{x \to 0} \frac{\sin x}{x + \tan x} = 0.5 = \frac{1}{2} \).
14. For \( f(x) = \frac{\sqrt{x - 4}}{x - 16} \):

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>0.123106</td>
<td>15</td>
<td>0.127017</td>
</tr>
<tr>
<td>16.5</td>
<td>0.124038</td>
<td>15.5</td>
<td>0.125992</td>
</tr>
<tr>
<td>16.1</td>
<td>0.124805</td>
<td>15.9</td>
<td>0.125196</td>
</tr>
<tr>
<td>16.05</td>
<td>0.124902</td>
<td>15.95</td>
<td>0.125098</td>
</tr>
<tr>
<td>16.01</td>
<td>0.124980</td>
<td>15.99</td>
<td>0.125020</td>
</tr>
</tbody>
</table>

It appears that \( \lim_{x \to 16} \frac{\sqrt{x - 4}}{x - 16} = 0.125 = \frac{1}{8} \).

15. For \( f(x) = \frac{\sqrt{x + 4} - 2}{x} \):

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.236068</td>
<td>-1</td>
<td>0.267949</td>
</tr>
<tr>
<td>0.5</td>
<td>0.242641</td>
<td>-0.5</td>
<td>0.258343</td>
</tr>
<tr>
<td>0.1</td>
<td>0.248457</td>
<td>-0.1</td>
<td>0.251582</td>
</tr>
<tr>
<td>0.05</td>
<td>0.249224</td>
<td>-0.05</td>
<td>0.250786</td>
</tr>
<tr>
<td>0.01</td>
<td>0.249844</td>
<td>-0.01</td>
<td>0.250156</td>
</tr>
</tbody>
</table>

It appears that \( \lim_{x \to 0} \frac{\sqrt{x + 4} - 2}{x} = 0.25 = \frac{1}{4} \).
16. For \( f(x) = \frac{\tan 3x}{\tan 5x} \):

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>±0.2</td>
<td>0.439279</td>
</tr>
<tr>
<td>±0.1</td>
<td>0.566236</td>
</tr>
<tr>
<td>±0.05</td>
<td>0.591893</td>
</tr>
<tr>
<td>±0.01</td>
<td>0.599680</td>
</tr>
<tr>
<td>±0.001</td>
<td>0.599997</td>
</tr>
</tbody>
</table>

It appears that \( \lim_{x \to 0} \frac{\tan 3x}{\tan 5x} = 0.6 = \frac{3}{5} \).
1. (a) \( \lim_{x \to a} [f(x) + h(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} h(x) = -3 + 8 = 5 \) 
(b) \( \lim_{x \to a} [f(x)]^2 = \left(\lim_{x \to a} f(x)\right)^2 = (-3)^2 = 9 \) 
(c) \( \lim_{x \to a} \sqrt[3]{h(x)} = \sqrt[3]{\lim_{x \to a} h(x)} = \sqrt[3]{8} = 2 \) 
(d) \( \lim_{x \to a} \frac{1}{f(x)} = \frac{1}{\lim_{x \to a} f(x)} = \frac{1}{-3} = -\frac{1}{3} \) 
(e) \( \lim_{x \to a} \frac{f(x)}{h(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} h(x)} = \frac{-3}{8} = -\frac{3}{8} \) 
(f) \( \lim_{x \to a} \frac{g(x)}{f(x)} = \frac{\lim_{x \to a} g(x)}{\lim_{x \to a} f(x)} = 0 \) 
(g) The limit does not exist, since \( \lim_{x \to a} g(x) = 0 \) but \( \lim_{x \to a} f(x) \neq 0 \). 
(h) \( \lim_{x \to a} \frac{2f(x)}{h(x) - f(x)} = \frac{2\lim_{x \to a} f(x)}{\lim_{x \to a} h(x) - \lim_{x \to a} f(x)} = \frac{2(-3)}{8 - (-3)} = -\frac{6}{11} \)

2. (a) \( \lim_{x \to 2} [f(x) + g(x)] = \lim_{x \to 2} f(x) + \lim_{x \to 2} g(x) = 2 + 0 = 2 \) 
(b) \( \lim_{x \to 1} g(x) \) does not exist since its left- and right-hand limits are not equal, so the given limit does not exist. 
(c) \( \lim_{x \to 0} [f(x)g(x)] = \lim_{x \to 0} f(x) \cdot \lim_{x \to 0} g(x) = 0 \cdot 1.3 = 0 \) 
(d) Since \( \lim_{x \to 1} g(x) = 0 \) and \( g \) is in the denominator, but \( \lim_{x \to 1} f(x) = -1 \neq 0 \), the given limit does not exist. 
(e) \( \lim_{x \to 2} x^3 f(x) = \left[\lim_{x \to 2} x^3\right] \left[\lim_{x \to 2} f(x)\right] = 2^3 \cdot 2 = 16 \) 
(f) \( \lim_{x \to 1} \sqrt{3 + f(x)} = \sqrt{3 + \lim_{x \to 1} f(x)} = \sqrt{3 + 1} = 2 \)

3. \( \lim_{x \to -2} (3x^4 + 2x^2 - x + 1) = \lim_{x \to -2} 3x^4 + \lim_{x \to -2} 2x^2 - \lim_{x \to -2} x + \lim_{x \to -2} 1 \) [Limit Laws 1 and 2] 
\[= 3 \lim_{x \to -2} x^4 + 2 \lim_{x \to -2} x^2 - \lim_{x \to -2} x + \lim_{x \to -2} 1\] [3] 
\[= 3(-2)^4 + 2(-2)^2 - (-2) + (1)\] [9, 8, and 7] 
\[= 48 + 8 + 2 + 1 = 59\]

5. \( \lim_{x \to -8} (1 + \sqrt[3]{x}) (2 - 6x^2 + x^3) = \lim_{x \to -8} (1 + \sqrt[3]{x}) \cdot \lim_{x \to -8} (2 - 6x^2 + x^3) \) [Limit Law 4] 
\[= \left(\lim_{x \to -8} 1 + \lim_{x \to -8} \sqrt[3]{x}\right) \cdot \left(\lim_{x \to -8} 2 - 6 \lim_{x \to -8} x^2 + \lim_{x \to -8} x^3\right)\] [1, 2, and 3] 
\[= (1 + \sqrt[3]{-8}) \cdot (2 - 6 \cdot 8^2 + 8^3)\] [7, 10, 9] 
\[= (1 + 2) \cdot (2 - 6 \cdot 8^2 + 8^3) = (3)(130) = 390\]
7. \[ \lim_{x \to 1} \left( \frac{1 + 3x}{1 + 4x^2 + 3x^4} \right)^3 = \left( \lim_{x \to 1} \frac{1 + 3x}{1 + 4x^2 + 3x^4} \right)^3 \]
\[ = \left[ \frac{\lim_{x \to 1} (1 + 3x)}{\lim_{x \to 1} (1 + 4x^2 + 3x^4)} \right]^3 \]
\[ = \left[ \frac{1 + 3(1)}{1 + 4(1)^2 + 3(1)^4} \right]^3 = \left[ \frac{4}{8} \right]^3 = \left( \frac{1}{2} \right)^3 = \frac{1}{8} \]

[2, 1, and 3, 7, 8, and 9]

9. \[ \lim_{\theta \to \pi/2} \theta \sin \theta = \left( \lim_{\theta \to \pi/2} \theta \right) \left( \lim_{\theta \to \pi/2} \sin \theta \right) \]
\[ = \frac{\pi}{2} \cdot \sin \frac{\pi}{2} \]
\[ = \frac{\pi}{2} \]

[8 and Direct Substitution Property]

10. (a) The left-hand side of the equation is not defined for \( x = 2 \), but the right-hand side is.

(b) Since the equation holds for all \( x \neq 2 \), it follows that both sides of the equation approach the same limit as \( x \to 2 \), just as in Example 3. Remember that in finding \( \lim_{x \to a} f(x) \), we never consider \( x = a \).

12. \[ \lim_{x \to 4} \frac{x^2 + 5x + 4}{x^2 + 3x - 4} = \lim_{x \to 4} \frac{(x + 4)(x + 1)}{(x + 4)(x - 1)} = \lim_{x \to 4} \frac{x + 1}{x - 1} = \frac{-4 + 1}{-4 - 1} = -\frac{3}{5} = \frac{3}{5} \]

13. \[ \lim_{x \to 2} \frac{x^2 - x + 6}{x - 2} \] does not exist since \( x - 2 \to 0 \) but \( x^2 - x + 6 \to 8 \) as \( x \to 2 \).

15. \[ \lim_{t \to -3} \frac{t^2 - 9}{2t^2 + 7t + 3} = \lim_{t \to -3} \frac{(t + 3)(t - 3)}{(2t + 1)(t + 3)} = \lim_{t \to -3} \frac{t - 3}{2t + 1} = \frac{-3 - 3}{2(-3) + 1} = -\frac{6}{5} = \frac{6}{5} \]

19. By the formula for the sum of cubes, we have
\[ \lim_{x \to -2} \frac{x + 2}{x^3 + 8} = \lim_{x \to -2} \frac{x + 2}{(x + 2)(x^2 - 2x + 4)} = \lim_{x \to -2} \frac{1}{x^2 - 2x + 4} = \frac{1}{4 + 4 + 4} = \frac{1}{12}. \]

20. \[ \lim_{x \to 1} \frac{x^2 + 2x + 1}{x^4 - 1} = \lim_{x \to 1} \frac{(x + 1)^2}{(x^2 + 1)(x^2 - 1)} = \lim_{x \to -1} \frac{(x + 1)^2}{(x^2 + 1)(x + 1)(x - 1)} = \lim_{x \to 1} \frac{x + 1}{(x^2 + 1)(x - 1)} = \frac{0}{2(-2)} = 0 \]

Page 2
23. \[ \lim_{x \to -4} \frac{1 + \frac{1}{4}}{4 + x} = \lim_{x \to -4} \frac{x + 4}{4x} = \lim_{x \to -4} \frac{x + 4}{4x(4 + x)} = \lim_{x \to -4} \frac{1}{4x} = \frac{1}{4(-4)} = -\frac{1}{16} \]

24. \[ \lim_{t \to 0} \left( \frac{1}{t} - \frac{1}{t^2 + t} \right) = \lim_{t \to 0} \frac{(t^2 + t) - t}{t(t^2 + t)} = \lim_{t \to 0} \frac{t^2}{t(t + 1)} = \lim_{t \to 0} \frac{1}{t + 1} = 1 \]

28. Let \( f(x) = -\sqrt{x^3 + x^2}, \) \( g(x) = \sqrt{x^3 + x^2} \sin(\pi/x), \) and \( h(x) = \sqrt{x^3 + x^2}. \) Then \( -1 \leq \sin(\pi/x) \leq 1 \) \( \Rightarrow \)

\[ -\sqrt{x^3 + x^2} \leq \sqrt{x^3 + x^2} \sin(\pi/x) \leq \sqrt{x^3 + x^2} \Rightarrow \]

\( f(x) \leq g(x) \leq h(x). \) So since \( \lim_{x \to 0} f(x) = \lim_{x \to 0} h(x) = 0, \) by the Squeeze Theorem we have \( \lim_{x \to 0} g(x) = 0. \)

29. We have \( \lim_{x \to -4} (4x - 9) = 4(4) - 9 = 7 \) and \( \lim_{x \to -4} (x^2 - 4x + 7) = 4^2 - 4(4) + 7 = 7. \) Since \( 4x - 9 \leq f(x) \leq x^2 - 4x + 7 \) for \( x \geq 0, \lim_{x \to 4} f(x) = 7 \) by the Squeeze Theorem.

31. \(-1 \leq \cos(2/x) \leq 1 \) \( \Rightarrow \) \(-x^4 \leq x^4 \cos(2/x) \leq x^4. \) Since \( \lim_{x \to 0} (-x^4) = 0 \) and \( \lim_{x \to 0} x^4 = 0, \) we have \( \lim_{x \to 0} [x^4 \cos(2/x)] = 0 \) by the Squeeze Theorem.

32. \(-1 \leq \sin(2\pi/x) \leq 1 \) \( \Rightarrow \) \( 0 \leq \sin^2(2\pi/x) \leq 1 \) \( \Rightarrow \) \( 1 \leq 1 + \sin^2(2\pi/x) \leq 2 \) \( \Rightarrow \)

\( \sqrt{x} \leq \sqrt{x} \left[ 1 + \sin^2(2\pi/x) \right] \leq 2\sqrt{x}. \) Since \( \lim_{x \to 0^+} \sqrt{x} = 0 \) and \( \lim_{x \to 0^+} 2\sqrt{x} = 0, \) we have \( \lim_{x \to 0^+} \left[ \sqrt{x} \left( 1 + \sin^2(2\pi/x) \right) \right] = 0 \) by the Squeeze Theorem.

33. \[ |x - 3| = \begin{cases} 
    x - 3 & \text{if } x - 3 \geq 0 \\
    -(x - 3) & \text{if } x - 3 < 0 
\end{cases} = \begin{cases} 
    x - 3 & \text{if } x \geq 3 \\
    3 - x & \text{if } x < 3 
\end{cases} \]

Thus, \( \lim_{x \to 3^+} (2x + |x - 3|) = \lim_{x \to 3^+} (2x + x - 3) = \lim_{x \to 3^+} (3x - 3) = 3(3) - 3 = 6 \) and

\[ \lim_{x \to 3^-} (2x + |x - 3|) = \lim_{x \to 3^-} (2x + 3 - x) = \lim_{x \to 3^-} (x + 3) = 3 + 3 = 6. \] Since the left and right limits are equal, \( \lim_{x \to 3} (2x + |x - 3|) = 6. \)

35. Since \( |x| = -x \) for \( x < 0, \) we have \( \lim_{x \to 0^-} \left( \frac{1}{x} - \frac{1}{|x|} \right) = \lim_{x \to 0^-} \left( \frac{1}{x} - \frac{1}{-x} \right) = \lim_{x \to 0^-} \frac{2}{x}, \) which does not exist since the denominator approaches 0 and the numerator does not.
37. (a) (i) If \( x \to 1^+ \), then \( x > 1 \) and \( g(x) = x - 1 \). Thus, \( \lim_{x \to 1^+} g(x) = \lim_{x \to 1^+} (x - 1) = 1 - 1 = 0 \).

(ii) If \( x \to 1^- \), then \( x < 1 \) and \( g(x) = 1 - x^2 \). Thus, \( \lim_{x \to 1^-} g(x) = \lim_{x \to 1^-} (1 - x^2) = 1 - 1^2 = 0 \).

Since the left- and right-hand limits of \( g \) at 1 are equal, \( \lim_{x \to 1^-} g(x) = 0 \).

(iii) If \( x \to 0 \), then \(-1 < x < 1 \) and \( g(x) = 1 - x^2 \). Thus, \( \lim_{x \to 0} g(x) = \lim_{x \to 0} (1 - x^2) = 1 - 0^2 = 1 \).

(iv) If \( x \to -1^- \), then \( x < -1 \) and \( g(x) = -x \). Thus, \( \lim_{x \to -1^-} g(x) = \lim_{x \to -1^-} (-x) = -(-1) = 1 \).

(v) If \( x \to -1^+ \), then \(-1 < x < 1 \) and \( g(x) = 1 - x^2 \). Thus,

\[
\lim_{x \to -1^+} g(x) = \lim_{x \to -1^+} (1 - x^2) = 1 - (-1)^2 = 1 - 1 = 0
\]

(vi) \( \lim_{x \to -1} g(x) \) does not exist because the limits in part (iv) and part (v) are not equal.

(b)

55. Since the denominator approaches 0 as \( x \to -2 \), the limit will exist only if the numerator also approaches 0 as \( x \to -2 \). In order for this to happen, we need \( \lim_{x \to -2} (3x^2 + ax + a + 3) = 0 \)  \( \iff \)

\[
3(-2)^2 + a(-2) + a + 3 = 0 \iff 12 - 2a + a + 3 = 0 \iff a = 15.
\]

With \( a = 15 \), the limit becomes

\[
\lim_{x \to -2} \frac{3x^2 + 15x + 18}{x^2 + x - 2} = \lim_{x \to -2} \frac{3(x + 2)(x + 3)}{(x - 1)(x + 2)} = \lim_{x \to -2} \frac{3(x + 3)}{x - 1} = \frac{3(-2 + 3)}{-2 - 1} = \frac{3}{-3} = -1.
\]
1. From Definition 1, \( \lim_{x \to 4} f(x) = f(4) \).

3. (a) The following are the numbers at which \( f \) is discontinuous and the type of discontinuity at that number: \(-4\) (removable), \(-2\) (jump), \(2\) (jump), \(4\) (infinite).

(b) \( f \) is continuous from the left at \(-2\) since \( \lim_{x \to -2^-} f(x) = f(-2) \). \( f \) is continuous from the right at 2 and 4 since 
\[
\lim_{x \to -2^+} f(x) = f(2) \quad \text{and} \quad \lim_{x \to 4^+} f(x) = f(4).
\]
It is continuous from neither side at \(-4\) since \( f(-4) \) is undefined.

4. \( g \) is continuous on \([-4, -2), (-2, 2), [2, 4), (4, 6), \) and \((6, 8)\).

5. The graph of \( y = f(x) \) must have a discontinuity at \( x = 3 \) and must show that \( \lim_{x \to 3^-} f(x) = f(3) \).

6. 

9. Since \( f \) and \( g \) are continuous functions,
\[
\lim_{x \to 3} [2f(x) - g(x)] = 2 \lim_{x \to 3} f(x) - \lim_{x \to 3} g(x) \quad \text{[by Limit Laws 2 and 3]}
\]
\[
= 2f(3) - g(3) \quad \text{[by continuity of \( f \) and \( g \) at \( x = 3 \)]}
\]
\[
= 2 \cdot 5 - g(3) = 10 - g(3)
\]
Since it is given that \( \lim_{x \to 3} [2f(x) - g(x)] = 4 \), we have \( 10 - g(3) = 4 \), so \( g(3) = 6 \).
10. \( \lim_{{x \to 4}} f(x) = \lim_{{x \to 4}} \left( x^2 + \sqrt{7} - x \right) = \lim_{{x \to 4}} x^2 + \sqrt{\lim_{{x \to 4}} 7 - \lim_{{x \to 4}} x} = 4^2 + \sqrt{7 - 4} = 16 + \sqrt{3} = f(4). \)

By the definition of continuity, \( f \) is continuous at \( a = 4 \).

11. \( \lim_{{x \to -1}} f(x) = \lim_{{x \to -1}} (x + 2x^3)^4 = \left( \lim_{{x \to -1}} x + 2 \lim_{{x \to -1}} x^3 \right)^4 = [-1 + 2(-1)^3]^4 = (3)^4 = 81 = f(-1). \)

By the definition of continuity, \( f \) is continuous at \( a = -1 \).

12. For \(-4 < a < 4\) we have \( \lim_{{x \to a}} f(x) = \lim_{{x \to a}} x \sqrt{16 - x^2} = \lim_{{x \to a}} x \sqrt{\lim_{{x \to a}} 16 - \lim_{{x \to a}} x^2} = a \sqrt{16 - a^2} = f(a) \),

so \( f \) is continuous on \((-4, 4)\). Similarly, we get \( \lim_{{x \to 4^-}} f(x) = 0 = f(4) \) and \( \lim_{{x \to 4^+}} f(x) = 0 = f(-4) \),

so \( f \) is continuous from the left at 4 and from the right at \(-4\). Thus, \( f \) is continuous on \([-4, 4]\).

13. \( f(x) = -\frac{1}{(x - 1)^2} \) is discontinuous at 1 since \( f(1) \) is not defined.
14. \( f(x) = \begin{cases} 
\frac{1}{x-1} & \text{if } x \neq 1 \\
2 & \text{if } x = 1 
\end{cases} \) is discontinuous at 1 because \( \lim_{x \to 1} f(x) \) does not exist.

\[ 
\begin{array}{c}
\text{Graph of } f(x)
\end{array}
\]

15. \( f(x) = \begin{cases} 
1 - x^2 & \text{if } x < 1 \\
1/x & \text{if } x \geq 1 
\end{cases} \)

The left-hand limit of \( f \) at \( a = 1 \) is
\[ \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (1 - x^2) = 0. \]
The right-hand limit of \( f \) at \( a = 1 \) is
\[ \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (1/x) = 1. \]
Since these limits are not equal, \( \lim_{x \to 1} f(x) \) does not exist and \( f \) is discontinuous at 1.

\[ 
\begin{array}{c}
\text{Graph of } y = 1 - x^2
\end{array}
\]

16. \( f(x) = \begin{cases} 
\frac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\
1 & \text{if } x = 1 
\end{cases} \)

\[ \lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x^2 - x}{x^2 - 1} = \lim_{x \to 1} \frac{x(x - 1)}{(x + 1)(x - 1)} = \lim_{x \to 1} \frac{x}{x + 1} = \frac{1}{2}. \]

but \( f(1) = 1 \), so \( f \) is discontinuous at 1.

\[ 
\begin{array}{c}
\text{Graph of } y = \frac{1}{x}
\end{array}
\]

17. \( F(x) = \frac{x}{x^2 + 5x + 6} \) is a rational function. So by Theorem 5 (or Theorem 6), \( F \) is continuous at every number in its domain, \( \{ x \mid x^2 + 5x + 6 \neq 0 \} = \{ x \mid (x + 3)(x + 2) \neq 0 \} = \{ x \mid x \neq -3, -2 \} \) or \( (-\infty, -3) \cup (-3, -2) \cup (-2, \infty) \).

18. By Theorem 6, the root function \( \sqrt{x} \) and the polynomial function \( 1 + x^3 \) are continuous on \( \mathbb{R} \). By part 4 of Theorem 4, the product \( G(x) = \sqrt{x} (1 + x^3) \) is continuous on its domain, \( \mathbb{R} \).
19. By Theorem 5, the polynomials $x^2$ and $2x - 1$ are continuous on $(-\infty, \infty)$. By Theorem 6, the root function $\sqrt{x}$ is continuous on $[0, \infty)$. By Theorem 8, the composite function $\sqrt{2x - 1}$ is continuous on its domain, $[\frac{1}{2}, \infty)$. By part 1 of Theorem 4, the sum $R(x) = x^2 + \sqrt{2x - 1}$ is continuous on $[\frac{1}{2}, \infty)$. 

20. By Theorem 6, the trigonometric function $\sin x$ and the polynomial function $x + 1$ are continuous on $\mathbb{R}$. By part 5 of Theorem 4, $h(x) = \frac{\sin x}{x + 1}$ is continuous on its domain, $\{x \mid x \neq -1\}$. 

21. By Theorem 6, the root function $\sqrt{x}$ and the trigonometric function $\sin x$ are continuous on their domains, $[0, \infty)$ and $(-\infty, \infty)$, respectively. Thus, the product $F(x) = \sqrt{x}\sin x$ is continuous on the intersection of those domains, $[0, \infty)$, by part 4 of Theorem 4.

31. $f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ cx - 1 & \text{if } x \geq 2 \end{cases}$

$f$ is continuous on $(-\infty, 2)$ and $(2, \infty)$. Now $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (cx^2 + 2x) = 4c + 4$ and $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (cx - 1) = 8 - 2c$. So $f$ is continuous $\iff 4c + 4 = 8 - 2c \iff 6c = 4 \iff c = \frac{2}{3}$. Thus, for $f$ to be continuous on $(-\infty, \infty)$, $c = \frac{2}{3}$.

46. (a) $\lim_{x \to 0^+} F(x) = 0$ and $\lim_{x \to 0^-} F(x) = 0$, so $\lim_{x \to 0} F(x) = 0$, which is $F(0)$, and hence $F$ is continuous at $x = a$ if $a = 0$. For $a > 0$, $\lim_{x \to a} F(x) = \lim_{x \to a} x = a = F(a)$. For $a < 0$, $\lim_{x \to a} F(x) = \lim_{x \to a} (\frac{1}{x}) = -a = F(a)$. Thus, $F$ is continuous at $x = a$; that is, continuous everywhere.

(b) Assume that $f$ is continuous on the interval $I$. Then for $a \in I$, $\lim_{x \to a} |f(x)| = \left|\lim_{x \to a} f(x)\right| = |f(a)|$ by Theorem 8. (If $a$ is an endpoint of $I$, use the appropriate one-sided limit.) So $|f|$ is continuous on $I$.

(c) No, the converse is false. For example, the function $f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$ is not continuous at $x = 0$, but $|f(x)| = 1$ is continuous on $\mathbb{R}$.
1. (a) \( \lim_{x\to2} f(x) = \infty \)  
   (b) \( \lim_{x\to-1^-} f(x) = \infty \)  
   (c) \( \lim_{x\to-1^+} f(x) = -\infty \)  
   (d) \( \lim_{x\to\infty} f(x) = 1 \)  
   (e) \( \lim_{x\to\infty} f(x) = 2 \)  
   (f) Vertical: \( x = -1, x = 2 \); Horizontal: \( y = 1, y = 2 \)

2. (a) \( \lim_{x\to\infty} g(x) = 2 \)  
   (b) \( \lim_{x\to-\infty} g(x) = -2 \)  
   (c) \( \lim_{x\to3} g(x) = \infty \)  
   (d) \( \lim_{x\to0} g(x) = -\infty \)  
   (e) \( \lim_{x\to-2^+} g(x) = -\infty \)  
   (f) Vertical: \( x = -2, x = 0, x = 3 \); Horizontal: \( y = -2, y = 2 \)

3. \( f(0) = 0, \quad f(1) = 1, \)  
   \( \lim_{x\to\infty} f(x) = 0, \)  
   \( f \) is odd

4. \[ \text{Diagram showing the graph of } f(x) \]

5. \( \lim_{x\to2} f(x) = -\infty, \quad \lim_{x\to\infty} f(x) = \infty, \)  
   \( \lim_{x\to-\infty} f(x) = 0, \quad \lim_{x\to0^+} f(x) = \infty, \)  
   \( \lim_{x\to0^-} f(x) = -\infty \)
7. \( f(0) = 3, \quad \lim_{x \to 0^-} f(x) = 4, \)
\[ \lim_{x \to 0^+} f(x) = 2, \]
\[ \lim_{x \to -\infty} f(x) = -\infty, \quad \lim_{x \to 4^-} f(x) = -\infty, \]
\[ \lim_{x \to 4^+} f(x) = \infty, \quad \lim_{x \to -\infty} f(x) = 3 \]

8. \( \lim_{x \to 3} f(x) = -\infty, \quad \lim_{x \to \infty} f(x) = 2, \)
\[ f(0) = 0, \quad f \text{ is even} \]

11. Vertical: \( x \approx -1.62, x \approx 0.62, x = 1; \)
Horizontal: \( y = 1 \)
13. \[
\lim_{x \to -3^+} \frac{x + 2}{x + 3} = -\infty \text{ since the numerator is negative and the denominator approaches 0 from the positive side as } x \to -3^+. 
\]

15. \[
\lim_{x \to 1} \frac{2 - x}{(x - 1)^2} = \infty \text{ since the numerator is positive and the denominator approaches 0 through positive values as } x \to 1. 
\]

17. \[
\lim_{x \to (-\pi/2)^-} \sec x = \lim_{x \to (-\pi/2)^-} (1/\cos x) = -\infty \text{ since } \cos x \to 0 \text{ as } x \to (-\pi/2)^- \text{ and } \cos x < 0 \text{ for } -\pi < x < -\pi/2. 
\]

19. Divide both the numerator and denominator by \(x^3\) (the highest power of \(x\) that occurs in the denominator).

\[
\lim_{x \to \infty} \frac{x^3 + 5x}{2x^3 - x^2 + 4} = \lim_{x \to \infty} \frac{\frac{x^3 + 5x}{x^3}}{\frac{2x^3 - x^2 + 4}{x^3}} = \lim_{x \to \infty} \frac{1 + \frac{5}{x^2}}{2 - \frac{1}{x} + \frac{4}{x^3}} = \lim_{x \to \infty} \frac{1 + \frac{5}{x^2}}{\frac{2 - 0 + 4}{x^3}} = \lim_{x \to \infty} \frac{1 + \frac{5}{x^2}}{\frac{2 - 0 + 4}{x^3}} = \frac{1 + 5(0)}{2 - 0 + 4(0)} = \frac{1}{2} 
\]

21. First, multiply the factors in the denominator. Then divide both the numerator and denominator by \(u^4\).

\[
\lim_{u \to \infty} \frac{4u^4 + 5}{(u^2 - 2)(2u^2 - 1)} = \lim_{u \to \infty} \frac{4u^4 + 5}{2u^4 - 5u^2 + 2} = \lim_{u \to \infty} \frac{4u^4 + 5}{u^4} = \lim_{u \to \infty} \frac{4 + 5}{2 - \frac{5}{u^2} + \frac{2}{u^4}} 
\]

\[
= \lim_{u \to \infty} \frac{4 + 5}{\frac{2 - 5}{u^2} + \frac{2}{u^4}} = \lim_{u \to \infty} \frac{4 + 5}{2 - 5(0) + 2(0)} = \frac{4 + 5(0)}{2 - 0 + 4(0)} = \frac{4}{2} = 2 
\]

23. \[
\lim_{x \to \infty} \left(\sqrt{9x^2 + x} - 3x\right) = \lim_{x \to \infty} \frac{(\sqrt{9x^2 + x} - 3x)(\sqrt{9x^2 + 3x})}{\sqrt{9x^2 + x} + 3x} = \lim_{x \to \infty} \frac{(\sqrt{9x^2 + x})^2 - (3x)^2}{\sqrt{9x^2 + x} + 3x} 
\]

\[
= \lim_{x \to \infty} \frac{(9x^2 + x) - 9x^2}{\sqrt{9x^2 + x} + 3x} = \lim_{x \to \infty} \frac{x}{\sqrt{9x^2 + x} + 3x} \cdot \frac{1/x}{1/x} 
\]

\[
= \lim_{x \to \infty} \frac{x}{\sqrt{9x^2 + x} + 3x} = \lim_{x \to \infty} \frac{1}{\sqrt{9 + 1/x} + 3} = \frac{1}{\sqrt{9} + 3} = \frac{1}{3 + 3} = \frac{1}{6} 
\]

24. \[
\lim_{x \to \infty} \left(\sqrt{x^2 + ax} - \sqrt{x^2 + bx}\right) = \lim_{x \to \infty} \frac{(\sqrt{x^2 + ax} - \sqrt{x^2 + bx})(\sqrt{x^2 + ax} + \sqrt{x^2 + bx})}{\sqrt{x^2 + ax} + \sqrt{x^2 + bx}} 
\]

\[
= \lim_{x \to \infty} \frac{(x^2 + ax) - (x^2 + bx)}{\sqrt{x^2 + ax} + \sqrt{x^2 + bx}} = \lim_{x \to \infty} \frac{[(a - b)x]/x}{\sqrt{x^2 + ax} + \sqrt{x^2 + bx}} 
\]

\[
= \lim_{x \to \infty} \frac{a - b}{\sqrt{1 + a/x} + \sqrt{1 + b/x}} = \frac{a - b}{\sqrt{1 + 0} + \sqrt{1 + 0}} = \frac{a - b}{2} 
\]
25. \( \lim_{x \to \infty} \cos x \) does not exist because as \( x \) increases \( \cos x \) does not approach any one value, but oscillates between 1 and \(-1\).

27. \( \lim_{x \to \infty} (x - \sqrt{x}) = \lim_{x \to \infty} \sqrt{x} (\sqrt{x} - 1) = \infty \) since \( \sqrt{x} \to \infty \) and \( \sqrt{x} - 1 \to \infty \) as \( x \to \infty \).

29. \( \lim_{x \to -\infty} (x^4 + x^5) = \lim_{x \to -\infty} x^5 (\frac{1}{x} + 1) \) [factor out the largest power of \( x \)] = \(-\infty \) because \( x^5 \to -\infty \) and \( 1/x + 1 \to 1 \) as \( x \to -\infty \).