1. (a) (i) Using Definition 1,
\[
m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to -3} \frac{f(x) - f(-3)}{x - (-3)} = \lim_{x \to -3} \frac{(x^2 + 2x) - (-3)}{x - (-3)} = \lim_{x \to -3} \frac{(x + 3)(x - 1)}{x + 3} = \lim_{x \to -3} (x - 1) = -4
\]
(ii) Using Equation 2,
\[
m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} = \lim_{h \to 0} \frac{f(-3 + h) - f(-3)}{h} = \lim_{h \to 0} \frac{((-3 + h)^2 + 2(-3 + h)) - (-3)}{h}
\]
\[
= \lim_{h \to 0} \frac{9 - 6h + h^2 - 6 + 2h - 3}{h} = \lim_{h \to 0} \frac{h(h - 4)}{h} = \lim_{h \to 0} (h - 4) = -4
\]
(b) Using the point-slope form of the equation of a line, an equation of the tangent line is \( y - 3 = -4(x + 3) \). Solving for \( y \) gives us \( y = -4x - 9 \), which is the slope-intercept form of the equation of the tangent line.

(c)

![Graphs showing the tangent line and the point-slope form equation](image)

2. (a) (i) \( m = \lim_{x \to -1} \frac{f(x) - f(-1)}{x - (-1)} = \lim_{x \to -1} \frac{x^3 - (-1)}{x + 1} = \lim_{x \to -1} \frac{(x + 1)(x^2 - x + 1)}{x + 1} = \lim_{x \to -1} (x^2 - x + 1) = 3 \)

(ii) \( m = \lim_{h \to 0} \frac{f(-1 + h) - f(-1)}{h} = \lim_{h \to 0} \frac{(-1 + h)^3 - (-1)}{h} = \lim_{h \to 0} \frac{h^3 - 3h^2 + 3h - 1 + 1}{h} = \lim_{h \to 0} (h^2 - 3h + 3) = 3 \)

(b) \( y - (-1) = 3[x - (-1)] \iff y + 1 = 3x + 3 \iff y = 3x + 2 \)

(c)

![Graphs showing the tangent line and the slope-intercept equation](image)
8. (a) Using (1),

\[ m = \lim_{x \to a} \frac{1}{\sqrt{x} - \sqrt{a}} = \lim_{x \to a} \frac{\sqrt{a} - \sqrt{x}}{\sqrt{ax} (x - a)} = \lim_{x \to a} \frac{(\sqrt{a} - \sqrt{x})(\sqrt{a} + \sqrt{x})}{\sqrt{ax} (x - a)(\sqrt{a} + \sqrt{x})} \]

\[ = \lim_{x \to a} \frac{a - x}{\sqrt{ax} (x - a)(\sqrt{a} + \sqrt{x})} = \lim_{x \to a} \frac{-1}{\sqrt{ax} (\sqrt{a} + \sqrt{x})} = -\frac{1}{\sqrt{a^3/2}} \text{ or } -\frac{1}{2a^{-3/2}} \]

(b) At (1, 1): \( m = -\frac{1}{2} \), so an equation of the tangent line is

\[ y - 1 = -\frac{1}{2} (x - 1) \iff y = -\frac{1}{2} x + \frac{3}{2} \]

At \((4, \frac{3}{2})\): \( m = -\frac{1}{16} \), so an equation of the tangent line is

\[ y - \frac{3}{2} = -\frac{1}{16} (x - 4) \iff y = -\frac{1}{16} x + \frac{3}{4} \]

9. (a) Since the slope of the tangent at \( t = 0 \) is 0, the car’s initial velocity was 0.

(b) The slope of the tangent is greater at \( C \) than at \( B \), so the car was going faster at \( C \).

(c) Near \( A \), the tangent lines are becoming steeper as \( x \) increases, so the velocity was increasing, so the car was speeding up.

Near \( B \), the tangent lines are becoming less steep, so the car was slowing down. The steepest tangent near \( C \) is the one at \( C \), so at \( C \) the car had just finished speeding up, and was about to start slowing down.

(d) Between \( D \) and \( E \), the slope of the tangent is 0, so the car did not move during that time.

10. (a) Runner A runs the entire 100-meter race at the same velocity since the slope of the position function is constant.

Runner B starts the race at a slower velocity than runner A, but finishes the race at a faster velocity.

(b) The distance between the runners is the greatest at the time when the largest vertical line segment fits between the two graphs—this appears to be somewhere between 9 and 10 seconds.

(c) The runners had the same velocity when the slopes of their respective position functions are equal—this also appears to be at about 9.5 s. Note that the answers for parts (b) and (c) must be the same for these graphs because as soon as the velocity for runner B overtakes the velocity for runner A, the distance between the runners starts to decrease.
14. (a) The average velocity between times \( t \) and \( t + h \) is
\[
\frac{s(t+h) - s(t)}{(t+h) - t} = \frac{(t+h)^2 - 8(t+h) + 18 - (t^2 - 8t + 18)}{h} = \frac{t^2 + 2th + h^2 - 8t - 8h + 18 - t^2 + 8t - 18}{h} = \frac{2th + h^2 - 8h}{h} = (2t + h - 8) \text{ m/s.}
\]

(i) \([3, 4]: t = 3, h = 4 - 3 = 1\), so the average velocity is \(2(3) + 1 - 8 = -1\) m/s.
(ii) \([3.5, 4]: t = 3.5, h = 0.5\), so the average velocity is \(2(3.5) + 0.5 - 8 = -0.5\) m/s.
(iii) \([4, 5]: t = 4, h = 1\), so the average velocity is \(2(4) + 1 - 8 = 1\) m/s.
(iv) \([4, 4.5]: t = 4, h = 0.5\), so the average velocity is \(2(4) + 0.5 - 8 = 0.5\) m/s.

(b) \(v(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h} = \lim_{h \to 0} (2t + h - 8) = 2t - 8\), so \(v(4) = 0\).

15. \(g'(0)\) is the only negative value. The slope at \(x = 4\) is smaller than the slope at \(x = 2\) and both are smaller than the slope at \(x = 2\). Thus, \(g'(0) < 0 < g'(4) < g'(2) < g'(-2)\).

16. (a) Since \(g(5) = -3\), the point \((5, -3)\) is on the graph of \(g\). Since \(g'(5) = 4\), the slope of the tangent line at \(x = 5\) is 4.

Using the point-slope form of a line gives us \(y - (-3) = 4(x - 5)\), or \(y = 4x - 23\).

(b) Since \((4, 3)\) is on \(y = f(x)\), \(f(4) = 3\). The slope of the tangent line between \((0, 2)\) and \((4, 3)\) is \(\frac{1}{2}\), so \(f'(4) = \frac{1}{2}\).

17. We begin by drawing a curve through the origin with a slope of 3 to satisfy \(f(0) = 0\) and \(f'(0) = 3\). Since \(f'(1) = 0\), we will round off our figure so that there is a horizontal tangent directly over \(x = 1\). Last, we make sure that the curve has a slope of \(-1\) as we pass over \(x = 2\). Two of the many possibilities are shown.

23. Use Definition 4 with \(f(x) = 3 - 2x + 4x^2\).
\[
f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{[3 - 2(a+h) + 4(a+h)^2] - (3 - 2a + 4a^2)}{h} = \lim_{h \to 0} \frac{(3 - 2a - 2h + 4a^2 + 8ah + 4h^2) - (3 - 2a + 4a^2)}{h} = \lim_{h \to 0} \frac{-2h + 8ah + 4h^2}{h} = \lim_{h \to 0} \frac{h(-2 + 8a + 4h)}{h} = \lim_{h \to 0} (-2 + 8a + 4h) = -2 + 8a\]
Note that the answers to Exercises 29 – 34 are not unique.

29. By Definition 4, \( \lim_{h \to 0} \frac{(1 + h)^{10} - 1}{h} = f'(1) \), where \( f(x) = x^{10} \) and \( a = 1 \).

\( Or: \) By Definition 4, \( \lim_{h \to 0} \frac{(1 + h)^{10} - 1}{h} = f'(0) \), where \( f(x) = (1 + x)^{10} \) and \( a = 0 \).

Note that the answers to Exercises 29 – 34 are not unique.

30. By Definition 4, \( \lim_{h \to 0} \frac{\sqrt[4]{16 + h} - 2}{h} = f'(16) \), where \( f(x) = \sqrt[4]{x} \) and \( a = 16 \).

\( Or: \) By Definition 4, \( \lim_{h \to 0} \frac{\sqrt[4]{16 + h} - 2}{h} = f'(0) \), where \( f(x) = \sqrt[4]{16 + x} \) and \( a = 0 \).

Note that the answers to Exercises 29 – 34 are not unique.

31. By Equation 5, \( \lim_{x \to 5} \frac{2^x - 32}{x - 5} = f'(5) \), where \( f(x) = 2^x \) and \( a = 5 \).

Note that the answers to Exercises 29 – 34 are not unique.

32. By Equation 5, \( \lim_{x \to \pi/4} \frac{\tan x - 1}{x - \pi/4} = f'(\pi/4) \), where \( f(x) = \tan x \) and \( a = \pi/4 \).

35. The sketch shows the graph for a room temperature of 72\(^\circ\) and a refrigerator temperature of 38\(^\circ\). The initial rate of change is greater in magnitude than the rate of change after an hour.
38. (a) (i) \[ \frac{P(2002) - P(2000)}{2002 - 2000} = \frac{5886 - 3501}{2} = \frac{2385}{2} = 1192.5 \text{ locations/year} \]

(ii) \[ \frac{P(2001) - P(2000)}{2001 - 2000} = \frac{4709 - 3501}{1} = 1208 \text{ locations/year} \]

(iii) \[ \frac{P(2000) - P(1999)}{2000 - 1999} = \frac{3501 - 2135}{1} = 1366 \text{ locations/year} \]

(b) Using the values from (ii) and (iii), we have \[ \frac{1208 + 1366}{2} = 1287 \text{ locations/year} \]

(c) Estimating \( A \) as (1999, 2035) and \( B \) as (2001, 4960), the slope at 2000 is \[ \frac{4960 - 2035}{2001 - 1999} = \frac{2925}{2} = 1462.5 \text{ locations/year} \]

42. (a) \( f'(5) \) is the rate of growth of the bacteria population when \( t = 5 \) hours. Its units are bacteria per hour.

(b) With unlimited space and nutrients, \( f' \) should increase as \( t \) increases; so \( f'(5) < f'(10) \). If the supply of nutrients is limited, the growth rate slows down at some point in time, and the opposite may be true.

44. (a) \( f'(8) \) is the rate of change of the quantity of coffee sold with respect to the price per pound when the price is $8 per pound.

The units for \( f'(8) \) are pounds/(dollars/pound).

(b) \( f'(8) \) is negative since the quantity of coffee sold will decrease as the price charged for it increases. People are generally less willing to buy a product when its price increases.

47. Since \( f(x) = x \sin(1/x) \) when \( x \neq 0 \) and \( f(0) = 0 \), we have

\[ f'(0) = \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h} = \lim_{h \to 0} \frac{h \sin(1/h) - 0}{h} = \lim_{h \to 0} \sin(1/h). \] This limit does not exist since \( \sin(1/h) \) takes the values \(-1\) and \(1\) on any interval containing \(0\). (Compare with Example 5 in Section 1.3.)

48. Since \( f(x) = x^2 \sin(1/x) \) when \( x \neq 0 \) and \( f(0) = 0 \), we have

\[ f'(0) = \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h} = \lim_{h \to 0} \frac{h^2 \sin(1/h) - 0}{h} = \lim_{h \to 0} h \sin(1/h). \] Since \(-1 \leq \sin \frac{1}{h} \leq 1\), we have

\[-|h| \leq |h| \sin \frac{1}{h} \leq |h| \Rightarrow -|h| \leq h \sin \frac{1}{h} \leq |h|.\] Because \( \lim_{h \to 0} (-|h|) = 0 \) and \( \lim_{h \to 0} |h| = 0 \), we know that

\[ \lim_{h \to 0} \left( h \sin \frac{1}{h} \right) = 0 \] by the Squeeze Theorem. Thus, \( f'(0) = 0 \).
1. It appears that $f$ is an odd function, so $f'$ will be an even function—that is, $f'(-a) = f'(a)$.

(a) $f'(-3) \approx 1.5$  
(b) $f'(-2) \approx 1$
(c) $f'(-1) \approx 0$  
(d) $f'(0) \approx -4$
(e) $f'(1) \approx 0$  
(f) $f'(2) \approx 1$
(g) $f'(3) \approx 1.5$

2. Note: Your answers may vary depending on your estimates. By estimating the slopes of tangent lines on the graph of $f$, it appears that

(a) $f'(0) \approx -3$  
(b) $f'(1) \approx 0$
(c) $f'(2) \approx 1.5$  
(d) $f'(3) \approx 2$
(e) $f'(4) \approx 0$  
(f) $f'(5) \approx -1.2$

3. (a)' = II, since from left to right, the slopes of the tangents to graph (a) start out negative, become 0, then positive, then 0, then negative again. The actual function values in graph II follow the same pattern.

(b)' = IV, since from left to right, the slopes of the tangents to graph (b) start out at a fixed positive quantity, then suddenly become negative, then positive again. The discontinuities in graph IV indicate sudden changes in the slopes of the tangents.

(c)' = I, since the slopes of the tangents to graph (c) are negative for $x < 0$ and positive for $x > 0$, as are the function values of graph I.

(d)' = III, since from left to right, the slopes of the tangents to graph (d) are positive, then 0, then negative, then 0, then positive, then 0, then negative again, and the function values in graph III follow the same pattern.

Hints for Exercises 4–11: First plot $x$-intercepts on the graph of $f'$ for any horizontal tangents on the graph of $f$. Look for any corners on the graph of $f$—there will be a discontinuity on the graph of $f'$. On any interval where $f'$ has a tangent with positive (or negative) slope, the graph of $f''$ will be positive (or negative). If the graph of the function is linear, the graph of $f'$ will be a horizontal line.
Hints for Exercises 4–11: First plot $x$-intercepts on the graph of $f'$ for any horizontal tangents on the graph of $f$. Look for any corners on the graph of $f$ — there will be a discontinuity on the graph of $f'$. On any interval where $f$ has a tangent with positive (or negative) slope, the graph of $f'$ will be positive (or negative). If the graph of the function is linear, the graph of $f'$ will be a horizontal line.
Hints for Exercises 4–11: First plot $x$-intercepts on the graph of $f'$ for any horizontal tangents on the graph of $f$. Look for any corners on the graph of $f$—there will be a discontinuity on the graph of $f'$. On any interval where $f$ has a tangent with positive (or negative) slope, the graph of $f'$ will be positive (or negative). If the graph of the function is linear, the graph of $f'$ will be a horizontal line.
15. (a) By zooming in, we estimate that $f'(0) = 0$, $f'(\frac{1}{2}) = 1$, $f'(1) = 2$, and $f'(2) = 4$.
(b) By symmetry, $f'(-x) = -f'(x)$. So $f'\left(-\frac{1}{2}\right) = -1$, $f'(-1) = -2$, and $f'(-2) = -4$.
(c) It appears that $f'(x)$ is twice the value of $x$, so we guess that $f'(x) = 2x$.
(d) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{(x^2 + 2hx + h^2) - x^2}{h} = \lim_{h \to 0} \frac{2hx + h^2}{h} = \lim_{h \to 0} \frac{h(2x + h)}{h} = \lim_{h \to 0} (2x + h) = 2x$.

19. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\left[(x+h)^3 - 3(x+h) + 5\right] - (x^3 - 3x + 5)}{h} = \lim_{h \to 0} \frac{\left[(x^3 + 3x^2h + 3xh^2 + h^3) - 3x - 3h + 5\right] - (x^3 - 3x + 5)}{h} = \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 - 3h}{h} = \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 - 3h}{h} = \lim_{h \to 0} \frac{3x^2 + 3xh + h^2 - 3}{h} = \lim_{h \to 0} \frac{3x^2 + 3xh + h^2 - 3}{h} = \lim_{h \to 0} \frac{3x^2 + 3xh + h^2 - 3}{h} = 3x^2 - 3$

Domain of $f = \text{domain of } f' = \mathbb{R}$.

20. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\left(x + h + \sqrt{x+h}\right) - (x + \sqrt{x})}{h} = \lim_{h \to 0} \frac{\left(h + \frac{\sqrt{x+h} - \sqrt{x}}{h}\right) \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}}{h} = \lim_{h \to 0} \left[1 + \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})}\right] = \lim_{h \to 0} \left[1 + \frac{1}{\sqrt{x+h} + \sqrt{x}}\right] = 1 + \frac{1}{2\sqrt{x}}$

Domain of $f = [0, \infty)$, domain of $f' = (0, \infty)$.

21. $g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} \frac{\sqrt{1+2(x+h)} - \sqrt{1+2x}}{h} = \lim_{h \to 0} \frac{(1+2x+2h) - (1+2x)}{h \left[\sqrt{1+2(x+h)} + \sqrt{1+2x}\right]} = \lim_{h \to 0} \frac{2}{\sqrt{1+2(x+h)} + \sqrt{1+2x}} = \frac{2}{2\sqrt{1+2x}} = \frac{1}{\sqrt{1+2x}}$

Domain of $g = \left[-\frac{1}{2}, \infty\right)$, domain of $g' = \left(-\frac{1}{2}, \infty\right)$. 
23. \[ G'(t) = \lim_{h \to 0} \frac{G(t + h) - G(t)}{h} = \lim_{h \to 0} \frac{4(t + h) - 4t}{(t + h + 1)(t + 1)} = \lim_{h \to 0} \frac{4(t + h)(t + 1) - 4t(t + h + 1)}{h(t + h + 1)(t + 1)} \]

\[ = \lim_{h \to 0} \frac{(4t^2 + 4ht + 4t + 4h) - (4t^2 + 4ht + 4t)}{h(t + h + 1)(t + 1)} = \lim_{h \to 0} \frac{4h}{h(t + h + 1)(t + 1)} = \frac{4}{(t + 1)^2} \]

Domain of \( G \) = domain of \( G' = (-\infty, -1) \cup (-1, \infty). \)

27. \( f \) is not differentiable at \( x = -4 \), because the graph has a corner there, and at \( x = 0 \), because there is a discontinuity there.

28. \( f \) is not differentiable at \( x = 0 \), because there is a discontinuity there, and at \( x = 3 \), because the graph has a vertical tangent there.

29. \( f \) is not differentiable at \( x = -1 \), because the graph has a vertical tangent there, and at \( x = 4 \), because the graph has a corner there.

30. \( f \) is not differentiable at \( x = -1 \), because there is a discontinuity there, and at \( x = 2 \), because the graph has a corner there.

31. As we zoom in toward \((-1, 0)\), the curve appears more and more like a straight line, so \( f(x) = x + \sqrt{|x|} \) is differentiable at \( x = -1 \). But no matter how much we zoom in toward the origin, the curve doesn’t straighten out—we can’t eliminate the sharp point (a cusp). So \( f \) is not differentiable at \( x = 0 \).

33. \( a = f, b = f', c = f''. \) We can see this because where \( a \) has a horizontal tangent, \( b = 0 \), and where \( b \) has a horizontal tangent, \( c = 0 \). We can immediately see that \( c \) can be neither \( f \) nor \( f' \), since at the points where \( c \) has a horizontal tangent, neither \( a \) nor \( b \) is equal to 0.

34. Where \( d \) has horizontal tangents, only \( c \) is 0, so \( d' = c \). Curve \( c \) has negative tangents for \( x < 0 \) and \( b \) is the only graph that is negative for \( x < 0 \), so \( c' = b \). Curve \( b \) has positive tangents on \( \mathbb{R} \) (except at \( x = 0 \)), and the only graph that is positive on the same domain is \( a \), so \( b' = a \). We conclude that \( d = f, c = f', b = f'', \) and \( a = f''' \).
39. (a) Note that we have factored $x - a$ as the difference of two cubes in the third step.

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \frac{x^{1/3} - a^{1/3}}{x - a} = \lim_{x \to a} \frac{x^{1/3} - a^{1/3}}{(x^{1/3} - a^{1/3})(x^{2/3} + x^{1/3}a^{1/3} + a^{2/3})}$$

$$= \lim_{x \to a} \frac{1}{x^{2/3} + x^{1/3}a^{1/3} + a^{2/3}} = \frac{1}{3a^{2/3}} \text{ or } \frac{1}{3}a^{-2/3}$$

(b) $f'(0) = \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h} = \lim_{h \to 0} \frac{\sqrt[3]{h} - 0}{h} = \lim_{h \to 0} \frac{1}{h^{2/3}}$. This function increases without bound, so the limit does not exist, and therefore $f'(0)$ does not exist.

(c) $\lim_{x \to 0} |f'(x)| = \lim_{x \to 0} \frac{1}{3x^{2/3}} = \infty$ and $f$ is continuous at $x = 0$ (root function), so $f$ has a vertical tangent at $x = 0$.

40. (a) $g'(0) = \lim_{x \to 0} \frac{g(x) - g(0)}{x - 0} = \lim_{x \to 0} \frac{x^{2/3} - 0}{x} = \lim_{x \to 0} \frac{1}{x^{1/3}}$, which does not exist.

(b) $g'(a) = \lim_{x \to a} \frac{g(x) - g(a)}{x - a} = \lim_{x \to a} \frac{x^{2/3} - a^{2/3}}{x - a} = \lim_{x \to a} \frac{(x^{1/3} - a^{1/3})(x^{1/3} + a^{1/3})}{(x^{1/3} - a^{1/3})(x^{2/3} + x^{1/3}a^{1/3} + a^{2/3})}$

$$= \lim_{x \to a} \frac{x^{1/3} + a^{1/3}}{x^{2/3} + x^{1/3}a^{1/3} + a^{2/3}} = \frac{2a^{1/3}}{3a^{2/3}} = \frac{2}{3}a^{-1/3} \text{ or } \frac{2}{3}a^{-1/3}$$

(c) $g(x) = x^{2/3}$ is continuous at $x = 0$ and

$$\lim_{x \to 0} |g'(x)| = \lim_{x \to 0} \frac{2}{3|x|^{1/3}} = \infty.$$ This shows that $g$ has a vertical tangent line at $x = 0$. 
1. \( f(x) = 186.5 \) is a constant function, so its derivative is 0, that is, \( f'(x) = 0 \).

2. \( f(x) = \sqrt{30} \) is a constant function, so its derivative is 0, that is, \( f'(x) = 0 \).

3. \( f(x) = 5x - 1 \implies f'(x) = 5 - 0 = 5 \)

4. \( F(x) = -4x^{10} \implies F'(x) = -4(10x^{10-1}) = -40x^9 \)

5. \( f(x) = x^3 - 4x + 6 \implies f'(x) = 3x^2 - 4(1) + 0 = 3x^2 - 4 \)

6. \( f(t) = \frac{1}{2}t^6 - 3t^4 + t \implies f'(t) = \frac{1}{2}(6t^5) - 3(4t^3) + 1 = 3t^5 - 12t^3 + 1 \)

7. \( f(x) = x - 3 \sin x \implies f'(x) = 1 - 3 \cos x \)

8. \( y = \sin t + \pi \cos t \implies y' = \cos t + \pi(- \sin t) = \cos t - \pi \sin t \)

9. \( f(t) = \frac{1}{4}(t^4 + 8) \implies f'(t) = \frac{1}{4}(t^4 + 8)' = \frac{1}{4}(4t^{4-1} + 0) = t^3 \)

10. \( h(x) = (x - 2)(2x + 3) = 2x^2 - x - 6 \implies h'(x) = 2(2x) - 1 - 0 = 4x - 1 \)

11. \( y = x^{-2/5} \implies y' = -\frac{2}{5}x^{(-2/5)-1} = -\frac{2}{5}x^{-7/5} = -\frac{2}{5x^{7/5}} \)

12. \( R(t) = 5t^{-3/5} \implies R'(t) = 5\left[-\frac{3}{5}t^{(-3/5)-1}\right] = -3t^{-8/5} \)

13. \( V(r) = \frac{4}{3}\pi r^3 \implies V'(r) = \frac{4}{3}\pi(3r^2) = 4\pi r^2 \)

14. \( R(x) = \frac{\sqrt{10}}{x^7} = \sqrt{10}x^{-7} \implies R'(x) = -7\sqrt{10}x^{-8} = -\frac{7\sqrt{10}}{x^8} \)

15. \( F(x) = \left(\frac{1}{2}x\right)^5 = \left(\frac{1}{2}\right)^5x^5 = \frac{1}{32}x^5 \implies F'(x) = \frac{1}{32}(5x^4) = \frac{5}{32}x^4 \)

16. \( y = \sqrt{x}(x - 1) = x^{3/2} - x^{1/2} \implies y' = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2} = \frac{1}{2}x^{-1/2}(3x - 1) \) [factor out \(\frac{1}{2}x^{-1/2}\)]

or \( y' = \frac{3x - 1}{2\sqrt{x}} \).

17. \( y = 4\pi^2 \implies y' = 0 \) since \(4\pi^2\) is a constant.

18. \( g(u) = \sqrt{2}u + \sqrt{3}u = \sqrt{2}u + \sqrt{3}\sqrt{u} \implies g'(u) = \sqrt{2}(1) + \sqrt{3}\left(\frac{1}{2}u^{-1/2}\right) = \sqrt{2} + \sqrt{3}/(2\sqrt{u}) \)
19. \( y = \frac{x^2 + 4x + 3}{\sqrt{x}} = x^{3/2} + 4x^{1/2} + 3x^{-1/2} \quad \Rightarrow \quad y' = \frac{3}{2} x^{1/2} + 4 \left( \frac{1}{2} \right) x^{-1/2} + 3 \left( -\frac{1}{2} \right) x^{-3/2} = \frac{3}{2} \sqrt{x} + \frac{2}{2x \sqrt{x}} - \frac{3}{2x} \sqrt{x} \\ \text{[note that } x^{3/2} = x^{1/2} \cdot x^{1/2} = x \sqrt{x}] \)

20. \( y = \frac{x^2 - 2 \sqrt{x}}{x} = x - 2x^{-1/2} \quad \Rightarrow \quad y' = 1 - 2 \left( -\frac{1}{2} \right) x^{-3/2} = 1 + 1/(x \sqrt{x}) \)

21. \( v = t^2 - \frac{1}{\sqrt{t^3}} = t^2 - t^{-3/4} \quad \Rightarrow \quad v' = 2t - \left( -\frac{3}{4} \right) t^{-7/4} = 2t + \frac{3}{4 t^{7/4}} = 2t + \frac{3}{4t^{3/2}} \)

22. \( y = \frac{\sin \theta}{2} + \frac{c}{\theta} = \frac{1}{2} \sin \theta + c \theta^{-1} \quad \Rightarrow \quad y' = \frac{1}{2} \cos \theta + c(-1) \theta^{-2} = \frac{\cos \theta}{2} - \frac{c}{\theta^2} \)

23. \( z = \frac{A}{y^{10}} + B \cos y = Ay^{-10} + B \cos y \quad \Rightarrow \quad \frac{dz}{dy} = A(-10)y^{-11} + B(- \sin y) = -\frac{10A}{y^{11}} - B \sin y \)

24. \( u = \sqrt[3]{t^2} + 2 \sqrt[3]{t^3} = t^{2/3} + 2t^{3/2} \quad \Rightarrow \quad u' = \frac{2}{3} t^{-1/3} + 2 \left( \frac{3}{2} \right) t^{1/2} = \frac{2}{3 \sqrt[3]{t}} + 3 \sqrt{t} \)

25. \( y = 6 \cos x \quad \Rightarrow \quad y' = -6 \sin x. \) At \((\pi/3, 3), y' = -6 \sin(\pi/3) = -6 \sqrt{3}/2 = -3 \sqrt{3}\) and an equation of the tangent line is \( y - 3 = -3 \sqrt{3} (x - \pi/3) \) or \( y = -3 \sqrt{3}x + 3 + \pi \sqrt{3}. \) The slope of the normal line is \( 1/(3 \sqrt{3}) \) (the negative reciprocal of \(-3 \sqrt{3}\)) and an equation of the normal line is \( y - 3 = \frac{1}{3 \sqrt{3}} \left( x - \frac{\pi}{3} \right) \) or \( y = \frac{1}{3 \sqrt{3}}x + 3 - \frac{\pi}{9 \sqrt{3}}. \)

30. \( G(r) = \sqrt{r} + \frac{3}{\sqrt{r}} \quad \Rightarrow \quad G'(r) = \frac{1}{2} r^{-1/2} + \frac{1}{3} r^{-2/3} \quad \Rightarrow \quad G''(r) = -\frac{1}{4} r^{-3/2} - \frac{2}{9} r^{-5/3} \)

31. \( g(t) = 2 \cos t - 3 \sin t \quad \Rightarrow \quad g'(t) = -2 \sin t - 3 \cos t \quad \Rightarrow \quad g''(t) = -2 \cos t + 3 \sin t \)

33. \( \frac{d}{dx} (\sin x) = \cos x \quad \Rightarrow \quad \frac{d^2}{dx^2} (\sin x) = -\sin x \quad \Rightarrow \quad \frac{d^3}{dx^3} (\sin x) = -\cos x \quad \Rightarrow \quad \frac{d^4}{dx^4} (\sin x) = \sin x. \)

The derivatives of \( \sin x \) occur in a cycle of four. Since \( 99 = 4(24) + 3, \) we have \( \frac{d^9}{dx^9} (\sin x) = \frac{d^3}{dx^3} (\sin x) = -\cos x. \)

36. \( f(x) = x^3 + 3x^2 + x + 3 \) has a horizontal tangent when \( f'(x) = 3x^2 + 6x + 1 = 0 \quad \Leftrightarrow \quad x = \frac{-6 \pm \sqrt{36 - 12}}{6} = -1 \pm \frac{1}{3} \sqrt{6}. \)
41. (a) \( s = t^3 - 3t \Rightarrow v(t) = s'(t) = 3t^2 - 3 \Rightarrow a(t) = v'(t) = 6t \)
(b) \( a(2) = 6(2) = 12 \text{ m/s}^2 \)
(c) \( v(t) = 3t^2 - 3 = 0 \) when \( t^2 = 1 \), that is, \( t = 1 \) and \( a(1) = 6 \text{ m/s}^2 \).

49. (a) \( C(x) = 2000 + 3x + 0.01x^2 + 0.0002x^3 \Rightarrow C'(x) = 3 + 0.02x + 0.0006x^2 \)

(b) \( C'(100) = 3 + 0.02(100) + 0.0006(10,000) = 3 + 2 + 6 = \$11/\text{pair} \). \( C'(100) \) is the rate at which the cost is increasing as the 100th pair of jeans is produced. It predicts the cost of the 101st pair.

(c) The cost of manufacturing the 101st pair of jeans is \( C(101) - C(100) = (2000 + 303 + 102.01 + 206.0602) - (2000 + 300 + 100 + 200) = 11.0702 \approx \$11.07 \).

59. Let \( (a, a^2) \) be a point on the parabola at which the tangent line passes through the point \((0, -4)\). The tangent line has slope \(2a\) and equation \(y - (-4) = 2a(x - 0)\) \(\Leftrightarrow\ y = 2ax - 4\). Since \((a, a^2)\) also lies on the line, \(a^2 = 2a(a) - 4\), or \(a^2 = 4\).
So \(a = \pm 2\) and the points are \((2, 4)\) and \((-2, 4)\).

61. \( y = f(x) = ax^2 \Rightarrow f'(x) = 2ax \). So the slope of the tangent to the parabola at \(x = 2\) is \(m = 2a(2) = 4a\). The slope of the given line, \(2x + y = b\) \(\Leftrightarrow y = -2x + b\), is seen to be \(-2\), so we must have \(4a = -2\) \(\Leftrightarrow a = -\frac{1}{2}\). So when \(x = 2\), the point in question has \(y\)-coordinate \(-\frac{1}{2} \cdot 2^2 = -2\). Now we simply require that the given line, whose equation is \(2x + y = b\), pass through the point \((2, -2)\): \(2(2) + (-2) = b\) \(\Leftrightarrow b = 2\). So we must have \(a = -\frac{1}{2}\) and \(b = 2\).
1. Product Rule: \( y = (x^2 + 1)(x^3 + 1) \Rightarrow y' = (x^2 + 1)(3x^2) + (x^3 + 1)(2x) = 3x^4 + 3x^2 + 2x = 5x^4 + 3x^2 + 2x. \) 
Multiplying first: \( y = (x^2 + 1)(x^3 + 1) = x^5 + x^3 + x^2 + 1 \Rightarrow y' = 5x^4 + 3x^2 + 2x \) (equivalent). 

2. Quotient Rule: \( F(x) = \frac{x - 3x \sqrt{x}}{x^{1/2}} = \frac{x - 3x^{3/2}}{x^{1/2}} \Rightarrow \)
\[
F'(x) = \frac{x^{1/2} \left( 1 - \frac{9}{2} x^{1/2} \right) - \left( x - 3x^{3/2} \right) \left( \frac{1}{2} x^{-1/2} \right)}{\left( x^{1/2} \right)^2} = \frac{x^{1/2} - \frac{9}{2} x - \frac{3}{2} x^{1/2} + \frac{3}{2} x}{x} = \frac{\frac{1}{2} x^{1/2} - 3x}{x} = \frac{1}{2} x^{-1/2} - 3
\]
Simplifying first: \( F(x) = \frac{x - 3x \sqrt{x}}{x^{1/2}} = \sqrt{x} - 3x = x^{1/2} - 3x \Rightarrow F'(x) = \frac{1}{2} x^{-1/2} - 3 \) (equivalent).

For this problem, simplifying first seems to be the better method.

3. \( g(t) = t^3 \cos t \Rightarrow g'(t) = t^3(-\sin t) + (\cos t) \cdot 3t^2 = 3t^2 \cos t - t^3 \sin t \) or \( t^2(3 \cos t - t \sin t) \)

4. \( f(x) = \sqrt{x} \sin x \Rightarrow f'(x) = \sqrt{x} \cos x + \sin x \left( \frac{1}{2} x^{-1/2} \right) = \sqrt{x} \cos x + \frac{\sin x}{2 \sqrt{x}} \)

5. \( F(y) = \left( \frac{1}{y^2} - \frac{3}{y^4} \right) (y + 5y^3) = (y^{-2} - 3y^{-4})(y + 5y^3) \) \( \Rightarrow \)
\[
F'(y) = (y^{-2} - 3y^{-4})(1 + 15y^2) + (y + 5y^3)(-2y^{-3} + 12y^{-5})
\]
\[
= (y^{-2} + 15 - 3y^{-4} - 45y^{-2}) + (-2y^{-2} + 12y^{-4} - 10 + 60y^{-2})
\]
\[
= 5 + 14y^{-2} + 9y^{-4} \) or \( 5 + 14/y^2 + 9/y^4 \)

6. \( Y(u) = (u^{-2} + u^{-3})(u^5 - 2u^2) \) \( \Rightarrow \)
\[
Y'(u) = (u^{-2} + u^{-3})(5u^4 - 4u) + (u^5 - 2u^2)(-2u^{-3} - 3u^{-4})
\]
\[
= (5u^2 - 4u^{-1} + 5u - 4u^{-2}) + (-2u^{-2} - 3u + 4u^{-1} + 6u^{-2}) = 3u^2 + 2u + 2u^{-2}
\)

7. \( f(x) = \sin x + \frac{1}{2} \cot x \Rightarrow f'(x) = \cos x - \frac{1}{2} \csc^2 x \)

8. \( y = 2 \csc x + 5 \cos x \Rightarrow y' = -2 \csc x \cot x - 5 \sin x \)

9. \( h(\theta) = \theta \csc \theta - \cot \theta \Rightarrow h'(\theta) = \theta(-\csc \theta \cot \theta) + (\csc \theta) \cdot 1 - (-\csc^2 \theta) = \csc \theta - \theta \csc \theta \cot \theta + \csc^2 \theta \)

10. \( y = u(a \cos u + b \cot u) \Rightarrow \)
\[
y' = u(-a \sin u - b \csc^2 u) + (a \cos u + b \cot u) \cdot 1 = a \cos u + b \cot u - au \sin u - bu \csc^2 u
\]
11. \( g(x) = \frac{3x - 1}{2x + 1} \quad \Rightarrow \quad g'(x) = \frac{(2x + 1)(3) - (3x - 1)(2)}{(2x + 1)^2} = \frac{6x + 3 - 6x + 2}{(2x + 1)^2} = \frac{5}{(2x + 1)^2} \)

12. \( f(t) = \frac{2t}{4 + t^2} \quad \Rightarrow \quad f'(t) = \frac{(4 + t^2)(2) - (2t)(2t)}{(4 + t^2)^2} = \frac{8 + 2t^2 - 4t^2}{(4 + t^2)^2} = \frac{8 - 2t^2}{(4 + t^2)^2} \)

13. \( y = \frac{t^2}{3t^2 - 2t + 1} \quad \Rightarrow \quad y' = \frac{3t^2 - 2t + 1(2t) - t^2(6t - 2)}{(3t^2 - 2t + 1)^2} = \frac{2t[3t^2 - 2t + 1 - t(3t - 1)]}{(3t^2 - 2t + 1)^2} = \frac{2t(3t^2 - 2t + 1 - 3t^2 + t)}{(3t^2 - 2t + 1)^2} = \frac{2t(1 - t)}{(3t^2 - 2t + 1)^2} \)

14. \( y = \frac{t^3 + t}{t^4 - 2} \quad \Rightarrow \quad y' = \frac{(t^4 - 2)(3t^2 + 1) - (t^3 + t)(4t^3)}{(t^4 - 2)^2} = \frac{3t^6 + t^4 - 6t^2 - 2 - 4t^6 + 4t^4}{(t^4 - 2)^2} = \frac{-t^6 - 3t^4 - 6t^2 - 2 + t^6 + 3t^4 + 6t^2 + 2}{(t^4 - 2)^2} = -\frac{2t^6 - 3t^4 - 6t^2 - 2}{(t^4 - 2)^2} = -\frac{2t^6}{(t^4 - 2)^2} \)

15. \( y = \frac{v^3 - 2v\sqrt{v}}{v} = v^2 - 2\sqrt{v} = v^2 - 2v^{1/2} \quad \Rightarrow \quad y' = 2v - 2\left(\frac{1}{2}\right)v^{-1/2} = 2v - v^{-1/2} \)

16. \( y = \frac{\sqrt{x} - 1}{\sqrt{x} + 1} \quad \Rightarrow \quad y' = \frac{(\sqrt{x} + 1)\left(\frac{1}{2\sqrt{x}}\right) - (\sqrt{x} - 1)\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x} + 1)^2} = \frac{1}{2} + \frac{1}{2\sqrt{x}} - \frac{1}{2} + \frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{x}(\sqrt{x} + 1)^2} \)

17. \( y = \frac{r^2}{1 + \sqrt{r}} \quad \Rightarrow \quad y' = \frac{(1 + \sqrt{r})(2r) - r^2\left(\frac{1}{2}r^{-1/2}\right)}{(1 + \sqrt{r})^2} = \frac{2r + 2r^{3/2} - \frac{1}{2}r^{3/2}}{(1 + \sqrt{r})^2} = \frac{2r + \frac{3}{2}r^{3/2}}{(1 + \sqrt{r})^2} = \frac{r(4 + 3r^{1/2})}{2(1 + \sqrt{r})^2} \)

18. \( y = \frac{cx}{1 + cx} \quad \Rightarrow \quad y' = \frac{(1 + cx)(c) - (cx)(c)}{(1 + cx)^2} = \frac{c + c^2x - c^2x}{(1 + cx)^2} = \frac{c}{(1 + cx)^2} \)

19. \( y = \frac{x}{\cos x} \quad \Rightarrow \quad y' = \frac{(\cos x)(1) - (x)(-\sin x)}{\cos^2 x} = \frac{\cos x + x\sin x}{\cos^2 x} \)
20. \[ y = \frac{1 + \sin x}{x + \cos x} \quad \Rightarrow \quad y' = \frac{(x + \cos x)(\cos x) - (1 + \sin x)(1 - \sin x)}{(x + \cos x)^2} = \frac{x \cos x + \cos^2 x - (1 - \sin^2 x)}{(x + \cos x)^2} = \frac{x \cos x}{(x + \cos x)^2} \]

21. \[ f(\theta) = \frac{\sec \theta}{1 + \sec \theta} \quad \Rightarrow \quad f'(\theta) = \frac{(1 + \sec \theta)(\sec \theta \tan \theta) - (\sec \theta)(\sec \theta \tan \theta)}{(1 + \sec \theta)^2} = \frac{(\sec \theta \tan \theta)((1 + \sec \theta) - \sec \theta)}{(1 + \sec \theta)^2} = \frac{\sec \theta \tan \theta}{(1 + \sec \theta)^2} \]

22. \[ y = \frac{1 - \sec x}{\tan x} \quad \Rightarrow \quad y' = \frac{\tan x (-\sec x \tan x) - (1 - \sec x)(\sec^2 x)}{(\tan x)^2} = \frac{\sec x (-\tan^2 x - \sec x + \sec^2 x)}{\tan^2 x} = \frac{\sec x (1 - \sec x)}{\tan^2 x} \]

23. \[ y = \frac{\sin x}{x^2} \quad \Rightarrow \quad y' = \frac{x^2 \cos x - (\sin x)(2x)}{(x^2)^2} = \frac{x(x \cos x - 2 \sin x)}{x^4} = \frac{x \cos x - 2 \sin x}{x^3} \]

24. \[ y = \frac{u^6 - 2u^3 + 5}{u^2} = u^4 - 2u^2 + 5u^{-2} \quad \Rightarrow \quad y' = 4u^3 - 2 - 10u^{-3} = 2u^{-3}(2u^6 - u^3 - 5) = 2(2u^6 - u^3 - 5)/u^3 \]

25. \[ f(x) = \frac{x}{x + c/x} \quad \Rightarrow \quad f'(x) = \frac{(x + c/x)(1) - x(1 - c/x^2)}{(x + c/x)^2} = \frac{x + c/x - x + c/x}{(x + c/x)^2} = \frac{2c/x}{x^2 + c/x^2} \cdot \frac{x^2}{x^2} = \frac{2cx}{x^2 + c^2} \]

26. \[ f(x) = \frac{ax + b}{cx + d} \quad \Rightarrow \quad f'(x) = \frac{(cx + d)(a) - (ax + b)(c)}{(cx + d)^2} = \frac{acx + ad - acx - bc}{(cx + d)^2} = \frac{ad - bc}{(cx + d)^2} \]

27. \[ y = \frac{2x}{x^2 - 1} \quad \Rightarrow \quad y' = \frac{(x + 1)(2) - (2x)(1)}{(x + 1)^2} = \frac{2}{(x + 1)^2} \cdot \frac{x + 1}{x + 1} = \frac{2}{x^2 - 1}. \text{ At } (1, 1), y' = \frac{1}{2}, \text{ and an equation of the tangent line is } y - 1 = \frac{1}{2}(x - 1), \text{ or } y = \frac{1}{2}x + \frac{1}{2}. \]

30. \[ y = (1 + x) \cos x \quad \Rightarrow \quad y' = (1 + x)(- \sin x) + \cos x - 1. \text{ At } (0, 1), y' = 1, \text{ and an equation of the tangent line is } y - 1 = 1(x - 0), \text{ or } y = x + 1. \]

34. \[ f(x) = \sec x \quad \Rightarrow \quad f'(x) = \sec x \tan x \quad \Rightarrow \quad f''(x) = \sec x (\sec^2 x + \tan x) - (\sec x \tan x) = \sec x (\sec^2 x + \tan^2 x). \]

\[ f''(\frac{\pi}{4}) = \sqrt{2} \left[ (\sqrt{2})^2 + 1^2 \right] = \sqrt{2} \cdot 2 + 1 = 3 \cdot \sqrt{2} \]
36. Let \( f(x) = x \sin x \) and \( h(x) = \sin x \), so \( f(x) = x h(x) \). Then \( f'(x) = h(x) + x h'(x) \),
\[
f''(x) = h'(x) + h'(x) + x h''(x) = 2h'(x) + x h''(x),
\]
\[
f'''(x) = 2h''(x) + h''(x) + x h'''(x) = 3h''(x) + x h'''(x), \ldots, f^{(n)}(x) = nh^{(n-1)}(x) + x h^{(n)}(x).
\]
Since \( 34 = 4(8) + 2 \), we have \( h^{(34)}(x) = h^{(2)}(x) = \frac{d^2}{dx^2} (\sin x) = -\sin x \) and \( h^{(35)}(x) = -\cos x \).
Thus, \( \frac{d^{35}}{dx^{35}} (x \sin x) = 35h^{(34)}(x) + x h^{(35)}(x) = -35 \sin x - x \cos x \).

40. (a) \( g(x) = f(x) \sin x \implies g'(x) = f(x) \cos x + \sin x \cdot f'(x) \), so
\[
g'(\frac{\pi}{3}) = f(\frac{\pi}{3}) \cos \frac{\pi}{3} + \sin \frac{\pi}{3} \cdot f'(\frac{\pi}{3}) = 4 \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot (-2) = 2 - \sqrt{3}
\]
(b) \( h(x) = \frac{\cos x}{f(x)} \implies h'(x) = \frac{f(x) \cdot (-\sin x) - \cos x \cdot f'(x)}{[f(x)]^2} \), so
\[
h'(\frac{\pi}{3}) = \frac{f(\frac{\pi}{3}) \cdot (-\sin \frac{\pi}{3}) - \cos \frac{\pi}{3} \cdot f'(\frac{\pi}{3})}{[f(\frac{\pi}{3})]^2} = \frac{4 \left(-\frac{\sqrt{3}}{2}\right) - \left(-\frac{1}{2}\right) \cdot (-2)}{4^2} = \frac{-2\sqrt{3} + 1}{16} = \frac{1 - 2\sqrt{3}}{16}
\]

41. We are given that \( f(5) = 1, f'(5) = 6, g(5) = -3, \) and \( g'(5) = 2.\)

(a) \( (fg)'(5) = f(5)g'(5) + g(5)f'(5) = (1)(2) + (-3)(6) = 2 - 18 = -16 \)
(b) \( \left(\frac{f}{g}\right)'(5) = \frac{g(5) f'(5) - f(5) g'(5)}{[g(5)]^2} = \frac{(-3)(6) - (1)(2)}{(-3)^2} = -\frac{20}{9} \)
(c) \( \left(\frac{g}{f}\right)'(5) = \frac{f(5) g'(5) - g(5) f'(5)}{[f(5)]^2} = \frac{(1)(2) - (-3)(6)}{(1)^2} = 20 \)

43. (a) From the graphs of \( f \) and \( g \), we obtain the following values: \( f(1) = 2 \) since the point \((1, 2)\) is on the graph of \( f \);

\[ g(1) = 1 \text{ since the point } (1,1) \text{ is on the graph of } g; \quad f'(1) = 2 \text{ since the slope of the line segment between } (0,0) \text{ and } (2,4) \]

\[ \text{is } \frac{4 - 0}{2 - 0} = 2; \quad g'(1) = -1 \text{ since the slope of the line segment between } (-2,4) \text{ and } (2,0) \text{ is } \frac{0 - 4}{2 - (-2)} = -1. \]

Now \( u(x) = f(x)g(x) \), so \( u'(1) = f(1)g'(1) + g(1) f'(1) = 2 \cdot (-1) + 1 \cdot 2 = 0.\)

(b) \( v(x) = f(x)/g(x) \), so \( v'(5) = \frac{g(5) f'(5) - f(5) g'(5)}{[g(5)]^2} = \frac{2(-\frac{1}{2}) - 3 \cdot \frac{2}{3}}{2^2} = \frac{-\frac{8}{3}}{4} = -\frac{2}{3} \).
44. (a) \( P(x) = F(x)G(x) \), so \( \frac{dP}{dx} = \frac{dF}{dx}G(x) + \frac{dG}{dx}F(x) = 3 \cdot \frac{2}{4} + 2 \cdot 0 = \frac{3}{2} \).

(b) \( Q(x) = \frac{F(x)}{G(x)} \), so \( \frac{dQ}{dx} = \frac{G(x) \frac{dF}{dx} - F(x) \frac{dG}{dx}}{[G(x)]^2} = \frac{1 \cdot \frac{1}{4} - 5 \cdot \frac{2}{3}}{1^2} = \frac{1}{4} + \frac{10}{3} = \frac{43}{12} \)

46. (a) \( y = x^2 f(x) \) \( \Rightarrow \) \( y' = 2xf'(x) + f(x) \cdot 2x \)

(b) \( y = \frac{f(x)}{x^2} \) \( \Rightarrow \) \( y' = \frac{x^2 f'(x) - f(x) \cdot 2x}{(x^2)^2} = \frac{x f'(x) - 2 f(x)}{x^3} \)

(c) \( y = \frac{x^2}{f(x)} \) \( \Rightarrow \) \( y' = \frac{f(x) \cdot 2x - x^2 f'(x)}{[f(x)]^2} \)

(d) \( y = \frac{1 + xf(x)}{\sqrt{x}} \) \( \Rightarrow \)

\[
y' = \frac{\sqrt{x} \left[ x f'(x) + f(x) \right] - \left[ 1 + xf(x) \right] \frac{1}{2 \sqrt{x}}}{(\sqrt{x})^2} = \frac{x^{3/2} f'(x) + x^{1/2} f(x) - \frac{1}{2} x^{-1/2} - \frac{1}{2} x^{1/2} f(x)}{x} \cdot 2x^{1/2} = \frac{xf(x) + 2x^2 f'(x) - 1}{2x^{3/2}} \]

51. If \( y = f(x) = \frac{x}{x+1} \), then \( f'(x) = \frac{(x+1)(1) - x(1)}{(x+1)^2} = \frac{1}{(x+1)^2} \). When \( x = a \), the equation of the tangent line is

\[
y - \frac{a}{a+1} = \frac{1}{(a+1)^2}(x-a). \]This line passes through \((1,2)\) when \(2 - \frac{a}{a+1} = \frac{1}{(a+1)^2}(1-a) \) \( \Leftrightarrow \)

\[
2(a+1)^2 - a(a+1) = 1 - a \quad \Leftrightarrow \quad 2a^2 + 4a + 2 - a^2 - a - 1 + a = 0 \quad \Leftrightarrow \quad a^2 + 4a + 1 = 0.
\]

The quadratic formula gives the roots of this equation as \( a = \frac{-4 \pm \sqrt{4^2 - 4(1)(1)}}{2(1)} = \frac{-4 \pm \sqrt{12}}{2} = -2 \pm \sqrt{3}, \) so there are two such tangent lines. Since

\[
f\left(-2 \pm \sqrt{3}\right) = \frac{-2 \pm \sqrt{3}}{-2 \pm \sqrt{3} + 1} = \frac{-2 \pm \sqrt{3}}{-1 \pm \sqrt{3}} = \frac{2 \pm 2 \sqrt{3} \mp \sqrt{3} - 3}{1 - 3} = \frac{-1 \mp \sqrt{3}}{-2} = \frac{1 \mp \sqrt{3}}{2},
\]

the lines touch the curve at \( A\left(-2 + \sqrt{3}, \frac{1 - \sqrt{3}}{2}\right) \approx \left(-0.27, -0.37\right) \) and \( B\left(-2 - \sqrt{3}, \frac{1 + \sqrt{3}}{2}\right) \approx \left(-3.73, 1.37\right). \)
1. Let \( u = g(x) = 4x \) and \( y = f(u) = \sin u \). Then \( \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = (\cos u)(4) = 4\cos 4x \).

7. \( F(x) = \sqrt[4]{1 + 2x + x^3} = (1 + 2x + x^3)^{1/4} \Rightarrow \)

\[
F'(x) = \frac{1}{4}(1 + 2x + x^3)^{-3/4} \cdot \frac{d}{dx} (1 + 2x + x^3) = \frac{1}{4(1 + 2x + x^3)^{3/4}} \cdot (2 + 3x^2)
\]

\[
= \frac{2 + 3x^2}{4(1 + 2x + x^3)^{3/4}} = \frac{2 + 3x^2}{4 \sqrt[4]{(1 + 2x + x^3)^3}}
\]

8. \( F(x) = (x^2 - x + 1)^3 \Rightarrow F'(x) = 3(x^2 - x + 1)^2(2x - 1) \)

11. \( y = \cos(a^3 + x^3) \Rightarrow y' = -\sin(a^3 + x^3) \cdot 3x^2 \quad [a^3 \text{ is just a constant}] = -3x^2 \sin(a^3 + x^3) \)

15. \( g(x) = (1 + 4x)^5(3 + x - x^2)^8 \Rightarrow \)

\[
g'(x) = (1 + 4x)^5 \cdot 8(3 + x - x^2)^7(1 - 2x) + (3 + x - x^2)^8 \cdot 5(1 + 4x)^4 \cdot 4
\]

\[
= 4(1 + 4x)^4(3 + x - x^2)^7[(2(1 + 4x)(1 - 2x) + 5(3 + x - x^2))]
\]

\[
= 4(1 + 4x)^4(3 + x - x^2)^7[2 + 4x - 16x^2 + (15 + 5x - 5x^2)] = 4(1 + 4x)^4(3 + x - x^2)^7(17 + 9x - 21x^2)
\]

19. \( y = x^3 \cos nx \quad \Rightarrow \quad y' = x^3(-\sin nx)(n) + \cos nx \cdot 3x^2 = x^2(3 \cos nx - nx \sin nx) \)

21. \( y = \sin(x \cos x) \Rightarrow y' = \cos(x \cos x) \cdot [x(-\sin x) + \cos x \cdot 1] = (\cos x - x \sin x) \cos(x \cos x) \)

28. \( y = \tan^2(3\theta) = (\tan 3\theta)^2 \Rightarrow y' = 2(\tan 3\theta) \cdot \frac{d}{d\theta} (\tan 3\theta) = 2 \tan 3\theta \cdot \sec^2 3\theta \cdot 3 = 6 \tan 3\theta \sec^2 3\theta \)

29. \( y = \sin \sqrt{1 + x^2} \Rightarrow y' = \cos \sqrt{1 + x^2} \cdot \frac{1}{2}(1 + x^2)^{-1/2} \cdot 2x = (x \cos \sqrt{1 + x^2})/\sqrt{1 + x^2} \)

31. \( y = (1 + \cos^2 x)^6 \Rightarrow y' = 6(1 + \cos^2 x)^5 \cdot 2 \cos x (-\sin x) = -12 \cos x \sin x (1 + \cos^2 x)^5 \)

32. \( y = \cot(x^2) + \cot^2 x = \cot(x^2) + (\cot x)^2 \Rightarrow \)

\[
y' = -\csc^2(x^2) \cdot 2x + 2(\cot x)^1(-\csc^2 x) = -2x \csc^2(x^2) - 2 \cot x \csc^2 x \)
\]

37. \( y = \sin \left( \tan \sqrt{\sin x} \right) \Rightarrow \)

\[
y' = \cos \left( \tan \sqrt{\sin x} \right) \cdot \frac{d}{dx} \left( \tan \sqrt{\sin x} \right) = \cos \left( \tan \sqrt{\sin x} \right) \sec^2 \sqrt{\sin x} \cdot \frac{d}{dx} \left( \sin x \right)^{1/2}
\]

\[
= \cos \left( \tan \sqrt{\sin x} \right) \sec^2 \sqrt{\sin x} \cdot \frac{1}{2} (\sin x)^{-1/2} \cdot \cos x = \cos \left( \tan \sqrt{\sin x} \right) \left( \sec^2 \sqrt{\sin x} \right) \left( \frac{1}{2 \sqrt{\sin x}} \right) \cos x
\]
39. \( y = (1 + 2x)^9 \Rightarrow y' = 10(1 + 2x)^9 \cdot 2 = 20(1 + 2x)^9 \). At \((0, 1)\), \( y' = 20(1 + 0)^9 = 20 \), and an equation of the tangent line is \( y - 1 = 20(x - 0) \), or \( y = 20x + 1 \).

47. \( F(x) = f(g(x)) \Rightarrow F'(x) = f'(g(x)) \cdot g'(x) \), so \( F'(5) = f'(g(5)) \cdot g'(5) = f'(-2) \cdot 6 = 4 \cdot 6 = 24 \)

49. (a) \( h(x) = f(g(x)) \Rightarrow h'(x) = f'(g(x)) \cdot g'(x) \), so \( h'(1) = f'(g(1)) \cdot g'(1) = f'(2) \cdot 6 = 5 \cdot 6 = 30 \).
   (b) \( H(x) = g(f(x)) \Rightarrow H'(x) = g'(f(x)) \cdot f'(x) \), so \( H'(1) = g'(f(1)) \cdot f'(1) = g'(3) \cdot 4 = 9 \cdot 4 = 36 \).

55. \( r(x) = f(g(h(x))) \Rightarrow r'(x) = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x) \), so
   \[ r'(1) = f'(g(h(1))) \cdot g'(h(1)) \cdot h'(1) = f'(g(2)) \cdot g'(2) \cdot 4 = f'(3) \cdot 5 \cdot 4 = 6 \cdot 5 \cdot 4 = 120 \]

58. The use of \( D, D^2, \ldots, D^n \) is just a derivative notation (see text page 86). In general, \( Df(2x) = 2f'(2x) \), \( D^2 f(2x) = 4f''(2x), \ldots, D^n f(2x) = 2^n f^{(n)}(2x) \). Since \( f(x) = \cos x \) and \( 50 = 4(12) + 2 \), we have \( f^{(50)}(x) = f^{(2)}(x) = -\cos x \), so \( D^{50} \cos 2x = -2^{50} \cos 2x \).
1. (a) \[ \frac{d}{dx}(xy + 2x + 3x^2) = \frac{d}{dx}(4) \Rightarrow \frac{d}{dx}(x \cdot y' + y \cdot 1 + 2 + 6x) = 0 \Rightarrow xy' = -y - 2 - 6x \Rightarrow \]
\[ y' = \frac{-y - 2 - 6x}{x} \text{ or } y' = -6 - \frac{y + 2}{x}. \]
(b) \[ xy + 2x + 3x^2 = 4 \Rightarrow xy = 4 - 2x - 3x^2 \Rightarrow y = \frac{4 - 2x - 3x^2}{x} = \frac{4}{x} - 2 - 3x, \text{ so } y' = -\frac{4}{x^2} - 3. \]
(c) From part (a), \[ y' = \frac{-y - 2 - 6x}{x} = -\frac{(4/x - 2 - 3x) - 2 - 6x}{x} = -\frac{4/x - 3x}{x} = -\frac{4}{x^2} - 3. \]

3. \[ \frac{d}{dx}(x^3 + x^2 y + 4y^2) = \frac{d}{dx}(6) \Rightarrow 3x^2 + (x^2 y' + y \cdot 2x) + 8yy' = 0 \Rightarrow y^2 y' = -3x^2 - 2xy \Rightarrow \]
\[ y = \frac{3x^2 + 2xy}{x^2 + 8y} = -\frac{2x^2}{x^2 + 8y} \]

7. \[ \frac{d}{dx}(x^2 y^2 + x \sin y) = \frac{d}{dx}(4) \Rightarrow x^2 \cdot 2yy' + y^2 \cdot 2x + x \cos y \cdot y' + \sin y \cdot 1 = 0 \Rightarrow \]
\[ 2x^2 yy' + x \cos y \cdot y' = -2x^2 - \sin y \Rightarrow (2x^2 + x \cos y) y' = -2x^2 - \sin y \Rightarrow y' = -\frac{2x^2 y - \sin y}{2x^2 y + x \cos y} \]

8. \[ \frac{d}{dx}(1 + x) = \frac{d}{dx}[\sin(xy^2)] \Rightarrow 1 = \cos(xy^2)(x \cdot 2yy' + y^2 \cdot 1) \Rightarrow 1 = 2xy \cos(xy^2) y' + y^2 \cos(xy^2) \Rightarrow \]
\[ 1 - y^2 \cos(xy^2) = 2xy \cos(xy^2) y' \Rightarrow y' = \frac{1 - y^2 \cos(xy^2)}{2xy \cos(xy^2)} \]

10. \[ \frac{d}{dx}[y \sin(x^2)] = \frac{d}{dx}[x \sin(y^2)] \Rightarrow y \cos(x^2) \cdot 2x + \sin(x^2) \cdot y' = x \cos(y^2) \cdot 2yy' + \sin(y^2) \cdot 1 \Rightarrow \]
\[ y' \cdot [\sin(x^2) - 2xy \cos(y^2)] = \sin(y^2) - 2x \cos(x^2) \Rightarrow y' = \frac{\sin(y^2) - 2xy \cos(x^2)}{\sin(x^2) - 2x \cos(y^2)} \]

15. \[ \frac{d}{dx}\{f(x) + x^2[f(x)]^3\} = \frac{d}{dx}(10) \Rightarrow f'(x) + x^2 \cdot 3[f(x)]^2 \cdot f'(x) + [f(x)]^3 \cdot 2x = 0. \text{ If } x = 1, \text{ we have } \]
\[ f'(1) + 1^2 \cdot 3[f(1)]^2 \cdot f'(1) + [f(1)]^3 \cdot 2(1) = 0 \Rightarrow f'(1) + 1 \cdot 3 \cdot 2^2 \cdot f'(1) + 2^3 \cdot 2 = 0 \Rightarrow f'(1) + 12f'(1) = -16 \Rightarrow 13f'(1) = -16 \Rightarrow f'(1) = -\frac{16}{13}. \]

16. \[ \frac{d}{dx}[g(x) + x \sin g(x)] = \frac{d}{dx}(x^2) \Rightarrow g'(x) + x \cos g(x) \cdot g'(x) + \sin g(x) \cdot 1 = 2x. \text{ If } x = 0, \text{ we have } \]
\[ g'(0) + 0 + \sin g(0) = 2(0) \Rightarrow g'(0) + 0 = 0 \Rightarrow g'(0) + 0 = 0 \Rightarrow g'(0) = 0. \]

17. \[ x^2 + xy + y^2 = 3 \Rightarrow 2x + xy' + y \cdot 1 + 2yy' = 0 \Rightarrow xy' + 2yy' = -2x - y \Rightarrow y'(x + 2y) = -2x - y \Rightarrow \]
\[ y' = \frac{-2x - y}{x + 2y} \text{. When } x = 1 \text{ and } y = 1, \text{ we have } y' = \frac{-2 - 1}{1 + 2} \cdot 1 = -\frac{3}{3} = -1, \text{ so an equation of the tangent line is } \]
\[ y - 1 = -1(x - 1) \text{ or } y = -x + 2. \]
21. \[2(x^2 + y^2)^2 = 25(x^2 - y^2) \Rightarrow 4(x^2 + y^2)(2x + 2yy') = 25(2x - 2yy') \Rightarrow\]
\[4(x + yy')(x^2 + y^2) = 25(x - yy') \Rightarrow 4yy'(x^2 + y^2) + 25yy' = 25x - 4x(x^2 + y^2) \Rightarrow\]
\[y' = \frac{25x - 4x(x^2 + y^2)}{25y + 4y(x^2 + y^2)}.\]
When \(x = 3\) and \(y = 1\), we have \(y' = \frac{75 - 120}{25 + 40} = -\frac{45}{65} = -\frac{9}{13}\), so an equation of the tangent line is \(y - 1 = -\frac{9}{13}(x - 3)\) or \(y = -\frac{9}{13}x + \frac{40}{13}\).

23. \[9x^2 + y^2 = 9 \Rightarrow 18x + 2yy' = 0 \Rightarrow 2yy' = -18x \Rightarrow y' = -9x/y \Rightarrow\]
\[y'' = -9 \left( \frac{y \cdot 1 - x \cdot y'}{y^2} \right) = -9 \left( \frac{y - x(-9x/y)}{y^2} \right) = -9 \cdot \frac{y^2 + 9x^2}{y^3} = -9 \cdot \frac{9}{y^3} \text{ [since } x \text{ and } y \text{ must satisfy the original equation, } 9x^2 + y^2 = 9\text{]. Thus, } y'' = -81/y^3.]