Signals & systems

Input signals -> Dynamic system -> Output signals
Signal Classification

- Continuous signal

- Discrete signal
System classification

• **Finite-dimensional system** (lumped-parameters system described by differential equations)
  – Linear systems and nonlinear systems
  – Continuous time and discrete time systems
  – Time-invariant and time varying systems

• **Infinite-dimensional system** (distributed parameters system described by partial differential equations)
  – Power transmission line
  – Antennas
  – Heat conduction
  – Optical fiber etc….
Continuous-time system
---characterized by differential equations

Definition: input and output of the system are continuous functions of the continuous variable time.

Discrete-time system
---characterized by different equations

Definition: input and output of the system change at only discrete instants of time.
Linear time invariant (LTI)--continuous

Described by a linear differential equation in time domain can be transferred to linear algebra form by using Laplace transform.
nonlinear system

Continuous system

Ordinary Linear differential equation

\[ b_n \frac{d^n y}{dt^n} + \cdots + b_1 \frac{dy}{dt} + b_0 y = a_m \frac{d^m x}{dt^m} + \cdots + a_1 \frac{dx}{dt} + a_0 x \]

discrete system

\[ b_n y(k + n) + \cdots + b_1 y(k + 1) + b_0 y(k) = a_m x(k + m) + \cdots + a_1 x(k + 1) + a_0 x(k) \]

\( k \in \mathbb{Z} \)

Ordinary Linear difference equation
examples

\[ y^{(5)}(t) - 2y''(t) + 5y'(t) - y(t) = f''''(t) + f'''(t) - 2f(t) \]

Linear time-invariant

\[ y'''(t) + y''(t) + ty(t) = f(t) \]

Linear time-varying

\[ y''(t) + 2y(t)y'(t) + y(t) = f'(t) + 3f(t) \]

Nonlinear time-invariant

\[ t^2y''(t) + y^2(t) = f(t) \]

Nonlinear time-varying
Linear system

*Definition: A system is linear if superposition principle is satisfied*

Linearity: (a) homogeneous principle (multiplication)
(b) addition principle
(c) superposition principle => (a) + (b)

\[
x(t) = u.x_1(t) + v.x_2(t) \\
y(t) = u.y_1(t) + v.y_2(t)
\]
\begin{align*}
y_1^{(5)}(t) - 2y_1''(t) + 5y_1'(t) - y_1(t) &= f_1''(t) + f_1'(t) - 2f_1(t) \\
k[y_1^{(5)}(t) - 2y_1''(t) + 5y_1'(t) - y_1(t)] &= k[f_1''(t) + f_1'(t) - 2f_1(t)] \\
\Rightarrow [ky_1(t)]^{(5)} - 2[ky_1''(t)] + 5[ky_1'(t)] - ky_1(t) &= kf_1''(t) + kf_1'(t) - 2kf_1(t)
\end{align*}

Multiplication law satisfied

\begin{align*}
y_1^{(5)}(t) - 2y_1''(t) + 5y_1'(t) - y_1(t) &= f_1''(t) + f_1'(t) - 2f_1(t) \\
+ \\
y_2^{(5)}(t) - 2y_2''(t) + 5y_2'(t) - y_2(t) &= f_2''(t) + f_2'(t) - 2f_2(t) \\
\downarrow \\
[y_1^{(5)}(t) + y_2^{(5)}(t)] - 2[y_1''(t) + y_2''(t)] + 5[y_1'(t) + y_2'(t)] - [y_1(t) + y_2(t)] \\
= [f_1''(t) + f_2''(t)] + [f_1'(t) + f_2'(t)] - 2[f_1(t) + f_2(t)] \\
\downarrow \\
[y_1(t) + y_2(t)]^{(5)} - 2[y_1(t) + y_2(t)]'' + 5[y_1(t) + y_2(t)]' - [y_1(t) + y_2(t)] \\
= [f_1(t) + f_2(t)]'' + [f_1(t) + f_2(t)]'' - 2[f_1(t) + f_2(t)]
\end{align*}

Addition law satisfied
Nonlinear systems

Nonlinear system
Common nonlinear phenomena (1)

(1) Ideal relay

\[ \text{sgn}(u) = \begin{cases} 
1, & u > 0 \\
0, & u = 0 \\
-1, & u < 0 
\end{cases} \]

(2) Saturation

Example: amplifier

Ideal saturation:

\[ \text{sat}(u) = \begin{cases} 
u, & |u| \leq 1 \\
\text{sgn}(u), & |u| > 1 
\end{cases} \]
(2) Dead zone

Example: amplifier with low input signals
Common nonlinear phenomena (2)

(3) Back lash

Example:
1. Gear gap
2. Hysteresis loop

(4) Dead zone

Example: amplifier with low input signals
Common nonlinear phenomena (3)

(5) hysteresis

Example: window comparator/Schmitt trigger

(6) parabola
Nonlinear system examples

(1) Pendulum equation

\[
ml \ddot{\theta} + kl \dot{\theta} + mg \sin \theta = 0
\]

\[
ml \ddot{\theta} + kl \dot{\theta} + mg\theta = 0
\]  
For small \( \theta \)
(2) Nonlinear spring equation

\[ m\ddot{x} + B\dot{x} + kx + k'x^3 = 0 \]

**(a) hard spring**

- \( k' > 0 \)
- \( |x(t)| \downarrow \Rightarrow f \downarrow \)

**(b) soft spring**

- \( k' < 0 \)
- \( |x(t)| \downarrow \Rightarrow f \uparrow \)
(3) Jump resonance: nonlinear system with force

\[ m\ddot{x} + B\dot{x} + kx + k'x^3 = p\cos \omega t \]
(4) Harmonic oscillation

\[
\begin{align*}
  u(t) &= A_0 \sin \omega t \\
  y(t) &= A_o |G(j\omega)| \sin[\omega t + \angle G(j\omega)]
\end{align*}
\]

\[
\begin{align*}
  u(t) &= A_0 \sin \omega t \\
  y(t) &= A_1 \sin[\omega t + \theta_1] + A_2 \sin[2\omega t + \theta_2] \\
  &\quad + A_3 \sin[3\omega t + \theta_3] + \cdots
\end{align*}
\]

\[m\ddot{x} + B\dot{x} + kx + k'x^3 = p \cos \omega t\]
Example:

\[ u(t) = A_0 \sin \omega t \]

\[ y(t) = u^3(t) = (A_0 \sin \omega t)^3 \]

\[ = \frac{3}{4} A_0^3 \sin \omega t - \frac{1}{4} A_0^3 \sin 3\omega t \]

(5) Subharmonic oscillation

\[ u(t) = A_0 \sin \omega t \]

\[ y(t) = A_1 \sin(\omega t + \theta_1) + A_2 \sin(\frac{\omega t}{2} + \theta_2) + A_3 \sin(\frac{\omega t}{3} + \theta_3) + \cdots \]
(6) Limit cycle

Example: Van der Pol’s equation

\[ \ddot{x} + \varepsilon (x^2 - 1) \dot{x} + x = 0 \]

\[ x^2 \ll 1 \Rightarrow \varepsilon (x^2 - 1) < 0 \]

\[ x^2 \gg 1 \Rightarrow \varepsilon (x^2 - 1) > 0 \]

\( \varepsilon > 0 \)

Stable limit cycle
\[ \ddot{x} + \varepsilon(x^2 - 1)\dot{x} + x = 0 \]

Unstable limit cycle

\[ \varepsilon > 0 \]

\[ x^2 \ll 1 \Rightarrow \varepsilon(x^2 - 1) > 0 \]

\[ x^2 \gg 1 \Rightarrow \varepsilon(x^2 - 1) < 0 \]